Extension of the predictive policy to a series of mechanical systems

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Abstract

In the literature, a great interest is reserved to complex systems (i.e. serial or parallel or mixed systems), constituted by the interconnection of single elements. The evolution of system reliability depends on its structure as well as on the evolution of the reliability of its individual elements. Maintenance activities on systems strongly affect element aging and system's operating life. Preventive maintenance, for example, is used to increase system availability reducing, as a consequence, the probability of failure. Generally, maintenance plans are performed with respect to some criteria depending on cost or on reliability/availability requirements. Therefore, the optimum maintenance scheduling of a system can be based on the minimization of the total cost or on the maximization of its availability. Many Authors emphasize the requirement on system reliability. In [1], for example, the concept of reliability equivalence from simple series and parallel systems to some complex systems is presented and reliability equivalence factors of complex systems are obtained. One of the most critical problems in preventive maintenance is the determination of the optimum frequency to perform maintenance actions on systems, in order to ensure a pre-defined level of availability. In this paper the predictive maintenance policy, for a single element, is extended to a system constituted by two series elements, named A and B. The transition from a single unit to a series system is not immediate and presents a great number of problems. Actually, when a maintenance action is scheduled for a system of this kind, the decision maker must decide if it is more convenient (with respect to some chosen criterion) to intervene on element A or B or on both. The proposed methodology deals with this practical problem in the context of the predictive maintenance policy. Research on this topic is in a running state and the methodology is only theoretically presented.

1 Introduction

In the literature, a great interest is reserved to complex systems (i.e. serial or parallel or mixed systems) because real systems are constituted by the interconnection of single items. The evolution of system reliability depends on its structure as well as on the evolution of its elements reliability. Maintenance activities on systems affect strongly element aging and system's operating life. To increase system availability and to reduce the probability of a failure, preventive maintenance is used. Maintenance plans are performed with respect to some criteria depending, for example, on cost, reliability or availability requirements. Therefore, the optimum maintenance scheduling of a system can be based on the minimization of the total cost or on the maximization of its availability. In [1], for example, a set of equations are developed for a two identical component parallel redundant system to determine the optimum maintenance interval based on the system failure rate. Many Authors emphasize the requirement on system reliability. In [2], the concept of reliability equivalence from simple series and parallel systems to some complex systems is presented and reliability equivalence factors of complex systems are obtained. One of the most critical problems in the preventive maintenance plans is the determination of the optimum frequency to perform maintenance actions on systems, in order to ensure its availability. A solution for this problem is presented in [3] for equipment that exhibits linearly increasing hazard rate and constant repair rate. The proposed algorithm calculates the interval of time between preventive maintenance actions for each component, by minimizing the costs. Similarly, in [4] an algorithm is presented for the maintenance management of a series system based on preventive maintenance in order to guarantee a pre-determined reliability level. This papers proposes the extension to a series of two components of a predictive maintenance policy. With the development of electronic monitoring systems, the interest for the predictive maintenance [5, 6, 7, 8, 9] policy has significantly increased with respect to the traditional preventive procedure. Although the introduction of sensors and electronic equipment add new costs, process monitoring of a system item increases its exploitation during its useful life. Actually, while the preventive maintenance action is scheduled by considering only the a-priori information about the population which the item belongs to, the predictive one is supported by the additional information coming from the monitoring system that follows each item stochastic degradation process. In analogy with the single item case [10], the extended methodology is based on a Bayesian approach but, differently from the previous case, the maintenance scheduling algorithm is more complex. In fact, when a maintenance action is scheduled, the decision maker must decide if it is more convenient (with respect to some chosen criterion) to intervene on element A or B or on both. The proposed methodology deals with this practical problem by employing a cost criterion. The proposed methodology is only theoretically presented. In particular, the paper is organized as follows. Section 2 introduces the predictive maintenance policy for a two component series system by presenting the adopted assumptions and the tracking system procedure. In Section 3 the cost function, to be minimized for the maintenance scheduling, is introduced. Section 4 shows the proposed algorithm and, finally, in Section 5 conclusions are drawn.

2 Predictive policy for a two components series system

A series of two components is considered. Both elements in the system are supposed to be monitored and two assumptions are made, described hereafter. The degradation process is described by a first autoregressive model with drift (AR(1)) or non stationary random walk model (RWM) [10]:

$$y_j(t+dt) = y_j(t) + \gamma' dt + \varepsilon_j(t) \tag{1}$$

with $j = 1,2$ indicating the component in the system, y the physical parameter correlated to the wear, γ' the mean value of the increment rate, $\varepsilon(t)$ a white process normally distributed with zero-mean and variance σ_ε^2. Since the degradation process is realistically observed at regular times, say Δt, equation 1 can be discretized. Therefore, by setting,

$$\gamma = \int \gamma' dt = \gamma' \Delta t \tag{2}$$

equation 1, becomes for each component of the series,

$$y_{i+1} = y_i + \gamma + \varepsilon_{i+1} \tag{3}$$

The degradation path cannot be generally observed directly but through a monitoring system that supplies a variable m correlated to the real degradation path y. Let m_i represent the value of such variable at time t_i. By hypothesizing a linear transfer function for the monitoring system, the relation between m_i and t_i can be expressed by:

$$m_i = a + by_i + \delta_i \tag{4}$$

where a e b are the coefficients of the linear transformation and δ_i represents the total system error at time i [10]. Variable δ, assumed normal distributed with zero-mean and variance σ_δ^2, represents the *reading error*, i.e. the imperfection of the monitoring system;

A system is considered failed when its reliability is less than a pre-defined reliability threshold R^*:

$$R_S \leq R^* \tag{5}$$

In order to compute system reliability, it is necessary to calculate each component reliability in the system. To this purpose, a Bayesian approach is proposed [11]. By using Bayes' theorem, the probability of a hypothesis, that at the beginning is exclusively constituted by the a-priori information, is updated as more information becomes available. In this context, the drift γ in equation 3 was introduced to represent a constant physical phenomenon characterizing all the units belonging to the same population. Actually, since the degradation behaviour of each component is realistically different even among components belonging to the same population, as a result of specific geometric or metallurgical characteristics as well as different environmental and working conditions, variable γ is more properly to be considered as a stochastic variable whose outcomes characterize each specific item. By assuming for γ a normal distribution with mean μ_γ and variance σ_γ^2, each component j is characterized at the beginning by the same a-priori information, $\pi(\gamma_j) = \pi(\gamma)$. As data flows from the monitoring system, its mean and variance $\mu_{\gamma,j}$ and $\sigma_{\gamma,j}^2$, can be updated by Bayesian approach. More computational details can be found in [9];

At each time instant, system reliability is the product of components reliabilities in the series system. Computation of each component reliability is based on equations (19 and 20) presented for the single item case in [10].

2.1 The tracking system procedure

Maintenance activities must be planned in advance. Actually, a minimum *management time* is required, say T_{min}. Realistically, this time is used by the technical staff to prepare the plant and to manage the maintenance actions. In analogy with single monitored component case [10], at each acquisition time t_i, the monitoring system will provide for each element in the system information on the variable representative of its degradation level. Then, each item reliability, $R_j(t_i + T_{min})$, is computed at the future time $t_i + T_{min}$. Consequently, system reliability, R_S, at time $t_i + T_{min}$, is the product of the two computed reliabilities. While R_S is greater than the fixed threshold, R^*, it is possible to wait for another acquisition of the monitoring system and then to proceed for a further estimation. Otherwise, the current time is the *decision time* and the maintenance activity can be scheduled. However, differently from the single unit case, when a maintenance activity is scheduled, the decision maker must decide which element in the series needs to be maintained or, alternatively, evaluate the convenience to intervene on both. Therefore, if A and B indicate the two items in the system, three possible scenarios must be carefully evaluated:

- **Scenario A**:the maintenance action is exclusively performed on element A.
- **Scenario B**: the maintenance action is exclusively performed on element B.
- **Scenario AB**: the maintenance action will be performed on both elements.

Obviously, the convenience in choosing the more suitable scenario depends on some adopted criterion. In this paper an economic criterion is presented. To this purpose, a cost function is appropriately defined (see next Section 3) and, based on it, the following procedure can be followed.

Let t_i be the decision time, then a maintenance action will be scheduled at time $t_i + T_{min}$. At time t_i, the decision maker evaluates costs associated with each scenario before taking any decision. Let us suppose to consider the scenario A. In this case, it is hypothesized that a maintenance action will be performed at time $t_i + T_{min}$ on element A, whereas no action will be performed on element B that will continue to work with a reduced reliability.

Therefore, from time $t_i + T_{min}$ for the maintained component A, that will be considered as good as new, the only available information is the a-priori information of the population which belongs to. On the contrary, since no maintenance action will be performed on component B, data collected by monitoring it till time t_i can be used to forecast its degradation path. Then, at the decision time t_i, the decision maker can estimate for this scenario the system degradation path and, consequently, schedule the next maintenance action by determining time T^* when system reliability will be again less than the pre-defined threshold. The procedure is obviously repeated for each scenario. It is interesting to underline that the estimation of the next maintenance time T^* at time t_i is exclusively performed for the evaluation of the costs associated with the three scenarios in a specific time horizon going from the intervention time $t_i + T_{min}$ to the next maintenance action. Actually, after performing the maintenance activity at the scheduled time $t_i + T_{min}$, elements will be monitored and then a new decision can be taken on the basis of the real data flowing from the monitoring system. This implies that the new maintenance activity will be scheduled in a time not necessarily coincident with T^*. Anyway, since the degradation rates are different for the elements in the system, time T^* is generally different for each scenario. As a consequence, costs will be different.

3 Cost function

In order to build the cost function, the following considerations are taken into account. Costs associated with possible system failures, that can happen in the time horizon going from the scheduled maintenance time, $t_i + T_{min}$, to the next intervention time T^*, are considered. Let $[t_i, t_f]$ be a generic time interval, where t_i is the current time and t_f the future time ($t_f = t_i + T_{min}$) in which a maintenance action is scheduled . It is hypothesized that both elements are working at time t_i. Let indicate with T_{FS} the system failure time, with C_{UN} the system unavailability cost due to failure, with C_{PS} the cost for a maintenance action (depending on the specific element to be maintained in the system) and with C_{FS} the failure cost.

Since the time interval between t_i and $t_i + T_{min}$ is likely shorter than the time interval between two subsequent maintenance activities, failure probability will be here considered negligible. On the contrary, a failure event can happen with a certain probability in the interval $[t_i + T_{min}, T^*]$ or beyond T^*. In particular, two possible cases will be here taken into account:

1. A failure happens in the interval $[t_i + T_{min}, T^*]$ and is far from T^* **more than** T_{min}.
2. A failure happens in the interval $[t_i + T_{min}, T^2*]$ and is far from T^* **less than** T_{min}.

In the first case, system can be repaired before T^*. A failure cost C_{FS} and an unavailability cost $C_{UN} \cdot T_{min}$ will be paid. In both cases, since the scheduled time is $t_i + T_{min}$, the cost for a maintenance action on A or on B or on both elements, i.e. $C_{P(A,B,AB)}$, will be paid.

Therefore, the expected unitary global cost is defined as the rate between the expected cost and the expected employing time of the system:

$$C(t_i, T^*) = \frac{N_1 + N_2 + N_3}{D_1 + D_2 + D_3} \tag{6}$$

where:

$$\begin{aligned} N_1 &= C_{P(A,B,AB)} \\ N_2 &= (C_{FS} + C_{UN} T_{min}) P\{(t_i + T_{min}) < t_{gS} < (T^* - T_{min})\} \\ N_3 &= (C_{FS} + C_{UN}(T^* - \bar{t}_{gS2})) P\{(t^* + T_{min}) < t_{gS} < T^*\} \\ D_1 &= \bar{t}_{gS1} \\ D_2 &= \bar{t}_{gS2} \\ D_3 &= [T^* - (t_i + T_{min})] P\{t_{gs} > T^*\} \end{aligned} \tag{7}$$

In the previous equation:

- $\bar{t}_{gS1}$ represents the system mean failure time in the interval $[t_i + T_{min}, T^* - T_{min}]$.
- $\bar{t}_{gS2}$ represents the system mean failure time in the interval $[T^* - T_{min}, T^*]$.

The probabilities in equation 6 can be expressed as a function of system reliabilities. In particular:

$$P\{(t_i + T_{min}) < t_{gS} < (T^* - T_{min})\} = R_S(t_i + T_{min}) - R_S(T * - T_{min}) \tag{8}$$

$$P\{(T^* - T_{min}) < t_{gS} < T^*\} = R_S(T * - T_{min}) - R_S(T^*) \tag{9}$$

The expressions for the system mean failure time is:

$$\bar{t}_{gS1} = \frac{\int_{t_i + T_{min}}^{T^* - T_{min}} t f_S(t) dt}{\int_{t_i + T_{min}}^{T^* - T_{min}} f_S(t) dt} = \frac{N_4}{D_4} \tag{10}$$

where $f_S(t)$ represents the system failure time probability distribution function. Since $f_S = -R'_S(t)$, the numerator N_4 of the previous equation can be written as follows:

$$N_4 = -\int_{t_i+T_{min}}^{T^*-T_{min}} t dR_s(t) = (t_i+T_{min})R_S(t_i+T_{min}) - (t^*-T_{min})R_S(t^*-T_{min}) + \int_{t_i+T_{min}}^{T^*-T_{min}} R_S(t)dt \tag{11}$$

and the denominator D_4:

$$D_4 = R_S(t_i+T_{min}) - R_S(t^*-T_{min}) \tag{12}$$

Therefore, system mean failure time (eq. 10) becomes:

$$\bar{t}_{gS1} = \frac{(t_i+T_{min})R_S(t_i+T_{min}) - (t^*-T_{min})R_S(t^*-T_{min}) + \int_{t_i+T_{min}}^{T^*-T_{min}} R_S(t)dt}{R_S(t_i+T_{min}) - R_S(t^*-T_{min})} \tag{13}$$

By applying the same procedure for the computation of the system mean failure time in the time interval $[T^*-T_{min}, T^*]$, it follows:

$$\bar{t}_{gS2} = \frac{(T^*-T_{min})R_S(T^*-T_{min}) - (T^*)R_S(T^*) + \int_{T^*-T_{min}}^{T^*} R_S(t)dt}{R_S(T^*-T_{min}) - R_S(T^*)} \tag{14}$$

By employing the previous equations, the cost function in equation 6 can be easily estimated as a function of the computed system reliability. By simulation, it is possible to determine the intervention time $t_i + T_{min}$ and time T^* for each hypothesized scenario. Then, by equation 6, costs can be computed and the first decision (intervention on A or on B or on both) can be taken.

4 Algorithm

Two elements belonging to two different population are followed by an imperfect monitoring system.

1. At the current time t_i, system reliability $R_S(t_i+T_{min})$ is computed as the product of each monitored component reliabilities and compared with the pre-defined threshold.
   (a) If $R_S > R^*$ no decision is taken and a new estimation is performed with the next acquired data supplied by the monitoring system.
   (b) Else time t_i is the decision time and a maintenance action is scheduled at time $t_i + T_{min}$.
      i. For each scenario, estimate T^*
      ii. Compute costs by equation 6
      iii. Choose the scenario with the minimum cost.
2. After performing the maintenance action at time $t_i + T_{min}$, system will start a new cycle: the maintained element is considered as good as new, the other one will continue to work with a reduced reliability.
3. Repeat the process from the beginning.

5 Conclusions

This paper proposes the extension of a predictive maintenance policy procedure, presented in [10] for the case of a single monitored component, to a two component series system. In analogy with the single unit case, a Bayesian approach is proposed. However, after scheduling the maintenance activity, the decision maker must decide which element in the series needs to be maintained. To this purpose, a cost criterion is proposed by defining a cost function and hypothesizing three different scenarios. The procedure is only theoretically presented. Its efficacy will be tested in the future by simulation.

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