Statistical evidence of central moments as fault indicators in ball bearing diagnostics

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Abstract

This paper deals with post processing of vibration data coming from a experimental tests. An AC motor running at constant speed is provided with a faulted ball bearing, tests are done changing the type of fault (outer race, inner race and balls) and the stage of the fault (three levels of severity: from early to late stage). A healthy bearing is also measured for the aim of comparison. The post processing simply consists in the computation of scalar quantities that are used in condition monitoring of mechanical systems: variance, skewness and kurtosis. These are the second, the third and the fourth central moment of a real-valued function respectively. The variance is the expectation of the squared deviation of a random variable from its mean, the skewness is the measure of the lopsidedness of the distribution, while the kurtosis is a measure of the heaviness of the tail of the distribution, compared to the normal distribution of the same variance. Most of the papers in the last decades use them with excellent results. This paper does not propose a new fault detection technique, but it focuses on the informative content of those three quantities in ball bearing diagnostics from a statistical point of view. In this paper, a discriminant function analysis is used, to determine which central moment has a high discrimination power in the diagnostics of ball bearing in stationary conditions.

1 Introduction

Rotating components such as shafts, gears and bearings are widely used in Industry, aiming power distribution and minimizing mechanical losses in machineries. They are critical components in terms of loss of productivity in case of unexpected failures. As rotating machines they suffer fatigue cycles in working conditions, often aggravated by non-stationary loads. Academic research spent efforts in developing diagnostics tools to prevent and monitor the health status of these critical components in working conditions [1]. Actually several papers are available in literature, suggesting very powerful techniques but often requiring complex calculation and advanced skills in signal processing [2]. On the contrary most of Industries ask for quick indicators of the health status that simply measure how much the signal deviates from the ideal model. Different indicators have been developed specifically for gears [3], focusing on the increasing amplitude of specific spectrum components related to the rotating speed of the shaft. In the diagnostics of ball bearings, the use of statistical indicators is secondary compared with the large number of papers on signal processing, because of the more complex signal of a faulted bearing, which is a cyclostationary signal [4]. Nevertheless the use of central moments is the core part different techniques, such as the spectral kurtosis. This technique - adapted by Randall and Antoni to the bearing diagnostics [5] - ruled the research field from its introduction so far. De facto, the presence of impacts in ball bearing increases the kurtosis value of the vibration data. As a consequence, the kurtosis is used to highlight the most informative frequency band in the analysis of vibration data [6]. The 4th central moment is most used statistical parameter in bearing diagnostics but not the only. In the literature, other parameters such as the skewness (the 3rd central moment) has been proposed [7] but also more complex and powerful high order statistics [8]. On the other hand, the ISO guidelines simply suggest the root mean square

(RMS) of the vibration velocity, comparing the value with specific tables [9]. The use of a statistical parameter is hardly used as-is in diagnostics, but often different parameters are used at the same time [10, 11, 12] or the they are a part of a more complex algorithms [13] and both supervised [14] and unsupervised [15] expert systems. This paper presents a statistical analysis focused on the ability of the 2nd, 3rd and 4th central moment to discriminate different types of fault on ball bearings. Faults on outer race, inner race and a rotating element of a self-aligning bearing is been tested on a test rig. For each type of fault, three different levels of increasing damage has been tested. The collected data are compared with the results on a healthy bearing. A discriminant analysis returns the most promising set of statistical parameters in order to diagnose a faulted bearing. The results of this paper could be used as a starting point for more complex algorithms of condition monitoring. The paper is organized as follows: section 2 introduces the experimental setup and the different level of faults on the bearing, section 3 gives a short description of the statistical analysis used in the processing of the data. Section 4 reports the results of the statistical analysis and, finally, in section 5 conclusions are drawn.

2 Measurement set-up

The considered experimental set-up consists of a specific test rig designed to test double-row self-aligning ball bearings (e.g. FAG 1204). The bearing was mounted on a shaft that is driven by a transmission pulley directly connected to an asynchronous electric motor by a V-belt. Shaft rotation velocity was 1602 rpm, i.e. 26.7 Hz. The bearing was loaded by means of lever mechanisms and a radial load on the self-aligning bearings was set at 500 N to verify the effectiveness of the applied techniques in low load conditions. The test machine and the related accessories are illustrated in Fig. 1, whereas the main geometry of the bearing and the characteristic fault frequencies are reported in Table 1.

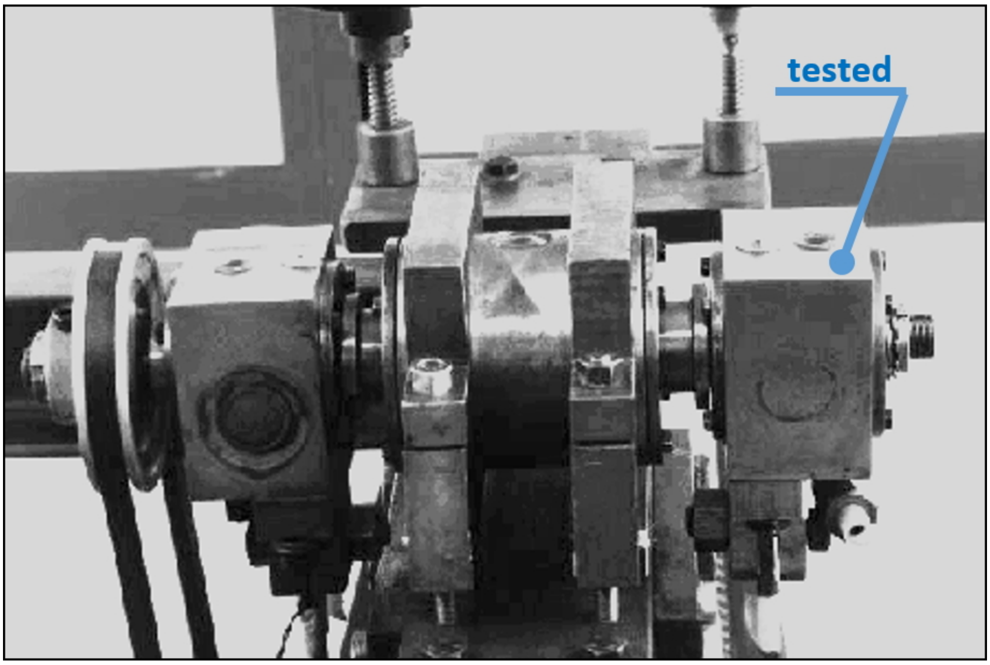

Figure 1: Test bench.

Bearing designation	1204
Pitch diameter [mm]	33.7
Ball diameter [mm]	6.35
Number of balls [per row]	12
Contact angle [deg]	0
Rotational frequency [Hz]	26.7

Table 1: Size and characteristic frequencies of the bearing

Each tested bearing was used to study only one kind of surface failure: one bearing was damaged on the inner race, one on the outer race and the last one on a rolling ball. Starting from the undamaged condition, three different pit dimensions were artificially produced by an electric pen to simulate a gradual increase of the damage. A total of 12 conditions were analyzed: one sound and three damaged conditions per tested bearing. A transversal line (approximately 1 mm wide) involving the race of one ball row was created on the race and - step by step - extended along the longitudinal direction to cover a square zone [Fig. 2 (c) and (f)]. In the case of the outer race, the bearing was mounted taking care to locate the damage at the point subjected to the highest load, where the probability of fault appearance is maximum. As far as the ball element damage is concerned, a round pit was produced: its radius was gradually increased from 1 mm [Fig. 2(g)] to cover, at the last stage, one hemisphere [Fig. 2(i)]. A radial acceleration signal was picked up from the top of the tested bearing casing by a B&K 4371 transducer, amplified and band-pass filtered by a B&K 2635 charge amplifier into the frequency range from 0.2 Hz to 30 kHz, and recorded on a TEAC Pcm Data Recorder. The sampling frequency used was 20 kHz and each measurement lasted 1 second, then the acquired signal $x[n]$ practically consists of 20 ksamples.

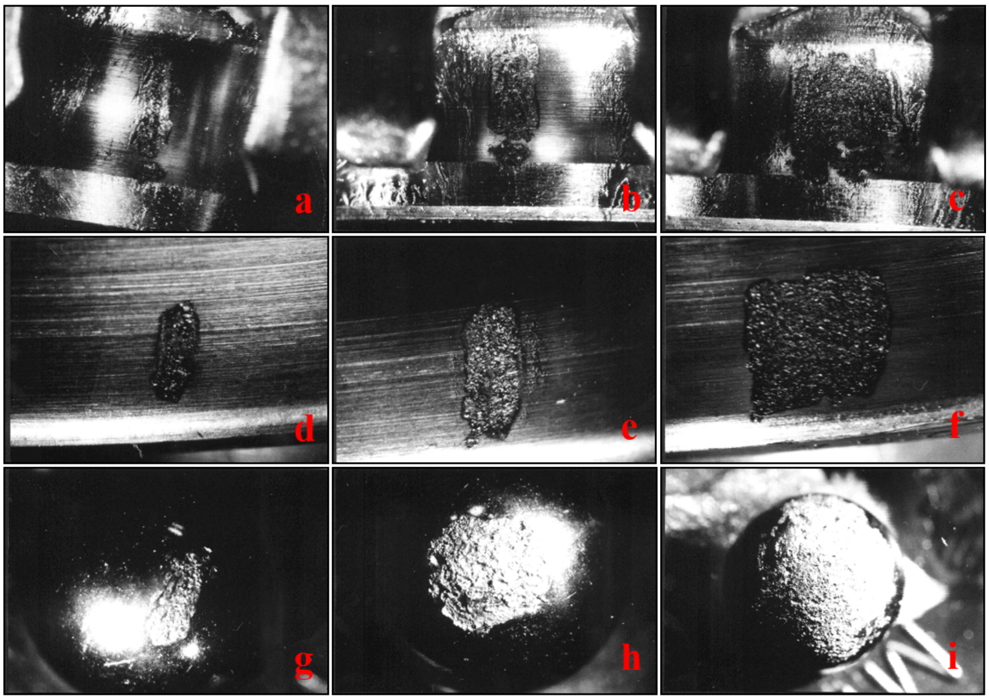

Figure 2: Bearing damages: location ad morphology of the surface faults to be tested, from early to distributed stage (a-c inner ring, d-f outer ring, g-i ball).

3 Methods

3.1 Central moments

In statistics, a central moment is the expected value of a specified integer power of the deviation of the random variable from the mean. The k-th central moment of a n-points data sample is:

$$m_k = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^k = E[(x-\bar{x})^k] \tag{1}$$

where $\bar{x}$ is the expected value of the variable x and $E[\cdot]$ is the expected value operator. The properties of a probability distribution of a data set can be characterized by the various moments, such as the spread and shape of the distribution. Strictly related to central moments are standardized moments, which are central moments, normalized by the standard deviation raised to the order of the considered moment. Usually, standardized moments are computed for higher moments (3rd order and more), while are not used for the first and second moments. For example, the standardized moment of the second order, i.e. the variance, is the constant unitary, since the standard deviation squared is the variance itself.

In this paper one central moment and two standardized moments are considered, but they are collectively referred to as central moment in the text, not to burdening the discussion. The selected central moments are detailed below. Moreover also the standard deviation is considered in this paper in order to test also the influence of the square root operator on the output results.

Variance

The variance is the second central moment of a data distribution and it measures how far a set of data are spread out from its mean.

$$var(x) = m_2 = E[(x-\overline{x})^2] \tag{2}$$

Skewness

The skewness is the third standardized moment of a data distribution and it is a measure of the asymmetry of the probability distribution of a real-valued data about its mean.

$$skew(x) = \frac{m_3}{m_2^{3/2}} = \frac{E[(x-\overline{x})^3]}{(E[(x-\overline{x})^2]^{3/2}} \tag{3}$$

Kurtosis

The kurtosis is the fourth standardized moment of a data distribution and it is a measure of the tailedness of the probability distribution of a real-valued data about its mean.

$$kurt(x) = \frac{m_4}{m_2^{4/2}} = \frac{E[(x-\overline{x})^4]}{(E[(x-\overline{x})^2]^{4/2}} \tag{4}$$

Standard deviation

The standard deviation is the square root of the variance and it a measure that is used to quantify the amount of dispersion of a set of data values.

$$std(x) = \sigma = \sqrt[2]{[(x-\overline{x})^2]} \tag{5}$$

3.2 The Mann-Whitney test

Mann-Whitney test is a nonparametric test that does not require the assumption of normal distribution [16]. It tests the null hypothesis that data in two data set X and Y are samples from continuous distributions with equal medians, against the alternative that they are not. The test assumes that the two samples are independent.It is nearly as efficient as the t-test for normal distributions.

Nonparametric methods are those data-analysis techniques that do not require the data to have a specific distribution. Nonparametric procedures may require one of the following two conditions:

- The data come from a symmetric distribution
- The data from two populations come from the same type of distribution

A general formulation is to assume that:

- The responses are ordinal.
- Under the null hypothesis H_0, the distributions of both populations are equal, i.e. $P(X > Y) = P(Y > X)$

- Under the alternative hypothesis H_1 the distributions of both populations are different $P(X > Y) \neq P(Y > X)$.

The Mann-Whitney test is included in most modern statistical packages. A description of the algorithm is out of the scopes of these paper, but it could be found in books of Statistics [17].

3.3 The discriminant analysis

A discriminant model [18, 19] among groups aims at predicting which group a new case belongs to. In most common applications of discriminant function analysis, many variables or predictors are considered in order to determine the ones with a high discrimination power. The linear discriminant function can be expressed by the following equation:

$$Z = a + W_1X_1 + W_2X_2 + \cdots + W_kX_k \tag{6}$$

where Z, the *discriminant score*, is used to predict group membership, a is the discriminant constant and are the explicative variables or predictors. In discriminant analysis the Total Sum of Squares (TSS) is partitioned into the Between Group (BSS) and the Within Group (WSS) sum of squares:

$$BSS = (\overline{Z}_0 - \overline{Z})^2 + (\overline{Z}_1 - \overline{Z})^2 = \sum_i (\overline{Z}_i - \overline{Z})^2 \tag{7}$$

$$WSS = (\overline{Z}_{i0} - \overline{Z}_0)^2 + (\overline{Z}_{i1} - \overline{Z}_1)^2 = \sum_j (\overline{Z}_{ij} - \overline{Z}_j)^2 \tag{8}$$

where i represents an individual case, j the group, Z_i an individual discriminant score, $\overline{Z}_j$ the mean discriminant score for group j (called *centroids*) and $\overline{Z}$ the grand mean of the discriminant scores. Discriminant analysis uses OLS to estimate the values of the parameters a and W_k that minimize the Within Group SS, WSS. In this study, a stepwise discriminant function analysis is applied. Then, the discrimination model is built step-by-step. At each step all variables are reviewed and evaluated to determine which one will contribute most to the discrimination among groups. That variable will then be included in the model, and the process starts again. Wilk's lambda criterion is used to select significant predictors. The latter indicates whether or not there is a significant relationship between the predictors and the dependent variable. To measure the *goodness-of-fit*, Wilk's lambda operates as follows. In case of two groups, the discriminant function can be extracted from data and the associated eigenvalue is:

$$\lambda = \frac{BSS}{WSS} \tag{9}$$

It turns out that if $\lambda = 0$ ($BSS = 0$), the model has no discriminatory power. The larger the value of λ, the greater the discriminatory power of the model. The Wilks' Λ for the discriminatory model is

$$\Lambda = \frac{1}{1+\lambda} = \frac{WSS}{TSS} \tag{10}$$

where Λ is chi-square distributed with $df = (k-1)$, where k is equal to the number of estimated parameters. Therefore, in terms of Λ, the more the parameter is close to 1 the less the discriminant power of the model is. For this reason Wilks'Λ is such an inverse quality criterion.

The stepwise introduction of predictors terminates when all the significant variables are considered and, of course, the discriminant power of the model is satisfying. Then an estimation of the *hit ratio* (HR) is needed. The latter gives the correctly classified observation units divided by the total number of observation units. As an example, consider a classification matrix for a two groups discriminant model, i.e. a $2x2$ matrix where the generic component $n_t p$ is the number of observations that belong to a t class while the predicted class is p. The following relation holds:

$$HR = \frac{n_{11} + n_{22}}{n_{11} + n_{12} + n_{21} + n_{22}} \tag{11}$$

When the same data set is used both for estimating the Discriminant analysis model and the classification, an over estimation of the HR is expected. To avoid this, the *leave-one-out* cross-validation method can be employed. This technique works by omitting each observation one at a time, recalculating the classification function using the remaining data, and then classifying the omitted observation.

4 Results

As described in section 2, for each specimen (ball bearing) 1 second of vibration signal has been acquired at 20kHz. The pre-processing of the data consists in a resampling of the signal, in order to have an integer and fixed number of sampling points for each revolution of the inner race of the bearing, i.e. for each rotation of the driving shaft. Since the number of test is limited to 10 specimens , each 1-second signal is splitted into single revolutions of the driving shaft. The shaft rotates at 27Hz, consequently each vibration data has been divided into 27 parts, each one corresponding to a single revolution of the shaft.

Figures 3, 4, 5 and 6 show the trend of statistical parameters for each shaft's revolution , each fault and and each level of damage available. It is interesting noting that an increase of the damage level does not obligatorily correspond to a higher value of statistical output.

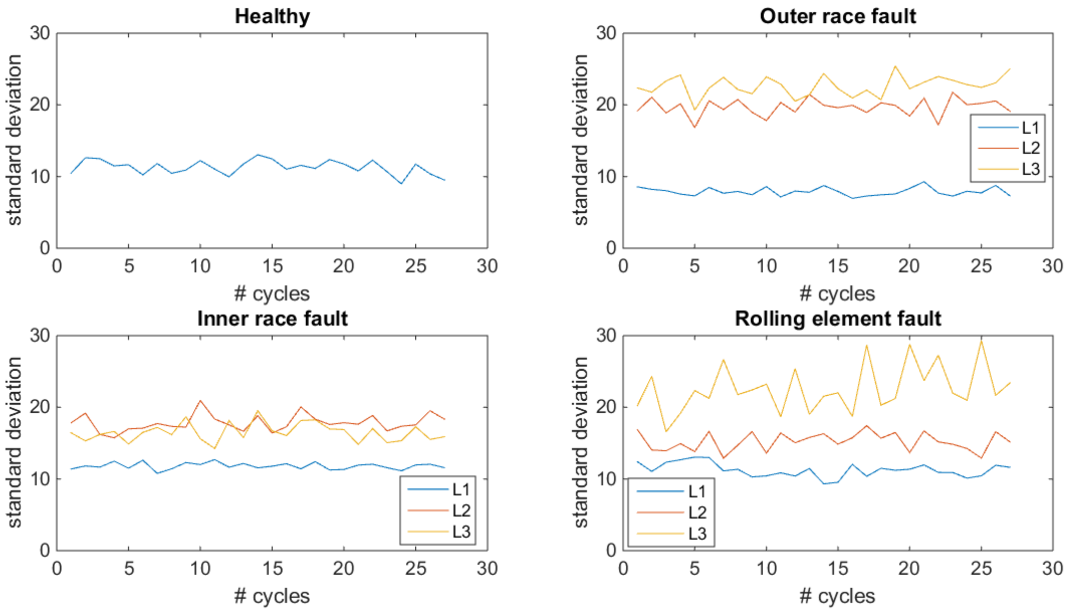

Figure 3: Standard deviation values for healthy and faulted bearings.

4.1 Mann-Whitney test

The Mann-Whitney test was used to compare the statistical evidence of kurtosis, skewness and variance as fault indicators. In particular, the test measures how far are the central moments for a faulted bearing compared to the values computed in a healthy bearing.

Kurtosis

- **Outer race fault**: significantly different from levels 2 and 3 (p-values: 0.0000). Not significantly different from level 1 (p-value: 0.4781).
- **Inner race fault**: significantly different from levels 1, 2 and 3 (p-values: 0.0000).
- **Rotating element fault**: not significantly different from levels 1 (p-value: 0.1882), 2 (p-value: 0.8732) and 3 (p-value: 0.2489).

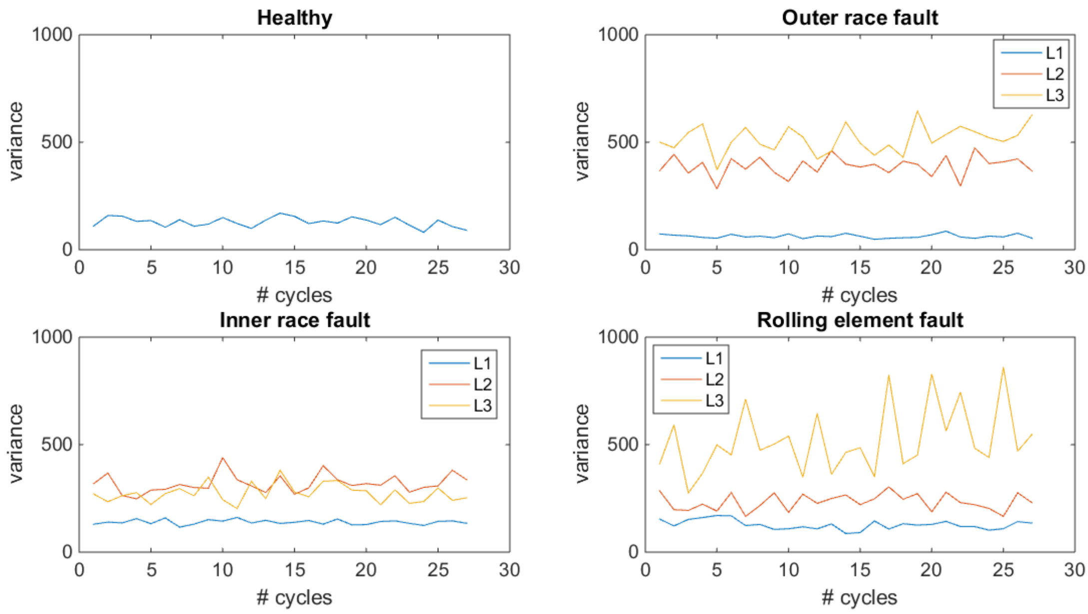

Figure 4: Variance values for healthy and faulted bearings.

Skewness

- **Outer race fault**: significantly different from levels 2 (p-value: 0.0025) and 3 (p-value: 0.0042). Not significantly different from level 1 (p-value: 0.0617).
- **Inner race fault**: significantly different from levels 1 (p-value: 0.0000). Not significantly different from levels 2 (p-value: 0.7819) and 3 (p-value: 0.0806).
- **Rotating element fault**: not significantly different from levels 1 (p-value: 0.4248), 2 (p-value: 0.8788) and 3 (p-value: 0.3393).

Variance and standard deviation

There is no relevant differences in the Mann-Whitney test between variance and standard deviation.

- **Outer race fault**: significantly different from levels 1, 2 and 3 (p-values: 0.0000).
- **Inner race fault**: significantly different from levels 2 and 3 (p-values: 0.0000). Not significantly different from level 1 (p-value: 0.0593).
- **Rotating element fault**: significantly different from levels 1 and 3 (p-values: 0.0000). Not significantly different from levels 2 (p-value: 0.9915).

4.2 Discriminant analysis

The discriminant analysis returns the optimal combination of the statistical parameters considered in order to maximize the ability of classification for each type of faulted bearings. The best results are the following:

Outer race fault

For the diagnostics of a bearing faulted on the outer race, the most discriminatory power is given by the use of **kurtosis, variance and standard deviation** at the same time. The total percentage of bearing correctly classified is 93.5%.

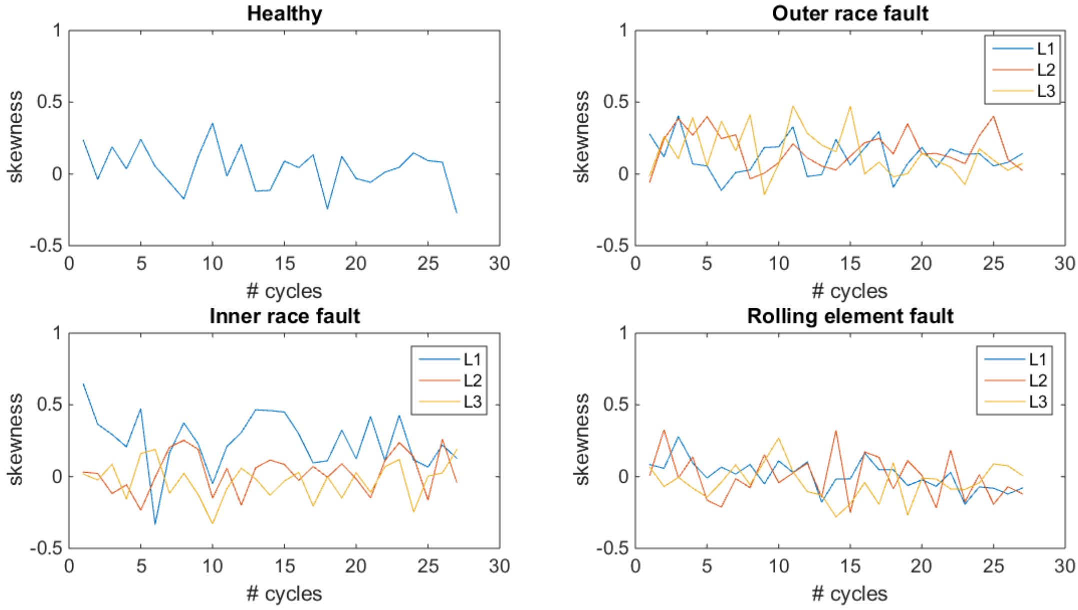

Figure 5: Skewness values for healthy and faulted bearings.

	Healthy	Level 1	Level 2	Level 3
Correct classification	96.3%	100%	92.6%	85.2%

Inner race fault

For the diagnostics of a bearing faulted on the inner race, the most discriminatory power is given by the use of **variance, standard deviation and kurtosis** at the same time. The total percentage of bearing correctly classified is 96.3%.

	Healthy	Level 1	Level 2	Level 3
Correct classification	100%	100%	100%	85.2%

Rotating element fault

For the diagnostics of a bearing faulted on a rotating element, the most discriminatory power is given by the use of **kurtosis, standard deviation, variance and kurtosis** at the same time. The total percentage of bearing correctly classified is 81.5%. It is worth noting that the case of the faulted rotating element is the most difficult to diagnose. Indeed, the kinematic behavior of the rotating elements that rotates on its own axis and also around the inner race, make the identification of a clear fault signature very difficult.

5 Conclusions

In this paper, four of the most common statistical parameters used in bearing diagnostics have been compared to assess their discriminant power on a real test rig. The statistical parameters considered are the standard deviation, the variance, the skewness and the kurtosis. Indeed the standard deviation and variance are related, nevertheless the aim of keeping both of them is to test also the influence of the power square operator on the output results. Three types of bearing faults at three different levels of wear have been considered, plus a brand-new healthy ball bearing as reference. Results show that the same statistical parameter is not the optimal choice for every type of faults, and that the level of wear could affect the results in different ways. In particular, the Mann-Whitney test on the data shows that:

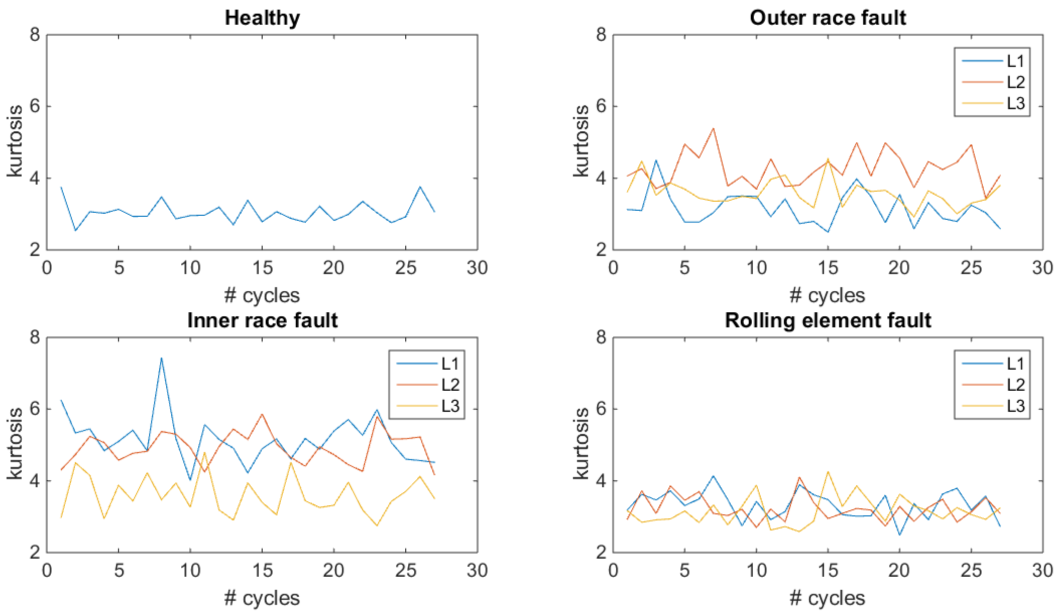

Figure 6: Kurtosis values for healthy and faulted bearings.

	Healthy	Level 1	Level 2	Level 3
Correct classification	77.8%	55.6%	96.3%	96.3%

- In case of a bearing faulted on the outer race, all the statistical parameters are able to identify the correct fault. At an early stage of the fault the kurtosis and the skewness are not significantly different from the corresponding values computed on a healthy bearing.
- In case of a bearing faulted on the inner race, the kurtosis is the best choice, while variance and standard deviation are good alternatives. The skewness is to be avoided.
- In case of a bearing faulted on a rotating element (i.e. a sphere), both kurtosis and skewness are to be avoided, while standard deviation and variance are good indicators.
- The use of standard deviation rather than the variance does not return sensible differences.

The discriminant analysis proved that the combined use of different statistical parameters may facilitate the identification of a specific type of damage on the rolling bearing. In particular the use of standard deviation, variance and kurtosis at the same time could lead to the 81.5% of positive results in the worse case, i.e. on the identification of a faulted rotating element. The same three variables allow the 93.5% and 96.3% of success in the case of a bearing faulted on the inner race and on the outer race, respectively. Indeed, this paper focuses on a single experimental campaign without any claim of generalization, that must be proved on several experiments.

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