Discrete Random Sampling: Theory and Practice in Machine Monitoring

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Abstract

Random sampling was first introduced in 1960's [1], and then reused in Compressive Sensing in 2004 [2] for having two important advantages: anti-aliasing property and low sampling rate, which decreases the demand of high speed in data acquisition and for big data storage. In this paper, a brief review is presented on the principal random sampling modes: Additive Random Sampling (ARS) and Jittered Random Sampling (JRS) that both use probability distributions: uniform or Gaussian [3-6]. In fact, the stationarity of the random point process is the essential criteria to guarantee the anti-aliasing property [7]. Accordingly, these modes of sampling are studied and used in simulation and hardware implementation. In addition, a study of the time quantization effect is done, to explore the different aspect of discrete random sampling [4, 5]. The performance of this sampling is approved by simulation and practice, using respectively Matlab and Arduino microcontroller for acquiring signals at a randomly spaced clock. A database was created in order to observe the impact of statistical parameters such as the mean and the standard deviation for Gaussian distribution, and the interval endpoints of uniform distribution. In conclusion, the ARS with uniform distribution, has been proved to have less limitations [4], where the spectrum of sampled signal is free of alias and the sampling frequency is reduced (a way smaller than Nyquist-Shannon sampling rate). To take advantage of this mode, it is applied on vibrational signals to enhance remoted machine monitoring.

Introduction

In these days, machine monitoring and supervision become one of the most important domain of research. Many axes of exploration are involved in this domain: signal processing, machine learning and too many others. Besides, industrial systems can now be remotely monitored because of the internet availability. In fact, as many other systems, machines can now be connected to any network by its Internet Protocol (IP) address due to the Internet of Things (IOT) concept. However, this combination is challenging in data acquisition and storage. In 2004, the compressive sensing was introduced to provide data with low rate in order to save energy consumption within wireless sensor networks [2]. Having a similar aspect, random sampling is found to be an advantageous way of acquiring data randomly with low frequency (much smaller than Nyquist rate) and guaranteeing a spectrum without aliasing. Though, these methods of sampling are not available, till now, by hardware means in markets. Thus, a brief study on the Random Sampling (RS) is presented: its concept, its impact on sampled signal and its implementation in hardware using Arduino microcontroller. In this article, a summary about the random sampling and its different modes is presented in the first section. In section 2, the conditions and the limitations of RS are cited. In section 3 a brief study on time quantization is given to define the Discrete Random Sampling, and section 4 summarizes the spectral analysis study. In sections 5 and 6 the application on simulated and real signals is shown. In the conclusion the article ends with a summary on the importance of random sampling.

1. Random Sampling and its Different Modes

Usually, samples are acquired from analogic signals at a constant rate, it is the well-known uniform sampling. Though, in some cases, some samples can't be acquired or may be missed, like in astronomy, structural and biomedical studies where the acquisition at some instants is impossible, so the time step between consecutive samples is not constant anymore, this type of sampling is defined as Non-Uniform Sampling, where the sampling frequency is not constant due to data loss or unavailability [12]. In fact, in the already cited cases, the non-uniform sampling (NUS) is a problem to be resolved, but in other cases, NUS is a chosen way to sample data in order to profit from its advantages. Actually, Random Sampling (RS) is a type of Non-Uniform sampling which is used in compressive sensing and Digital Alias-free Signal Processing (DASP) [8] for its advantages of free-aliasing and low sampling frequency in contrary with uniform sampling whose main condition is the Nyquist rate.

In time domain, the sampling process of a signal x(t) is modeled with a simple multiplication:

$$x_s(t) = x(t).s(t) \tag{1}$$

s(t), which is the sampling signal, is defined in (2):

$$S(t) = \sum_{n=-\infty}^{+\infty} \delta(t - t_n) \tag{2}$$

In uniform sampling the n^{th} instant of sampling is t_n=nT, while in random sampling t_n is a random variable that has a specific distribution law. The mode of random sampling is determined by the formula of t_n. The possible modes of RS are: Additive Random Sampling (ARS), Jittered Random Sampling (JRS), correlated Random Sampling (CRS) and Hybrid Additive Random Sampling (HARS). While, theoretically, the random variable may follow different probability distribution such as: exponential, uniform and Gaussian [3-6].

First, the additive random sampling (ARS) was first proposed by Shapiro and Silverman in [10] as a sampling method providing alias-free processing of analogic signals. As its name indicates, the sampling instant in this mode is obtained by adding a random variable to its previous:

$$t_n = t_{n-1} + \tau_n \qquad n = 0,1,2, \ldots \tag{3}$$

Where τ_n are independent and identically distributed (iid) variables with a probability density function (PDF) = p(τ) having a variance =σ^2 and a mean=μ. There are modified models of ARS that are introduced by [6] and [7]: Hybrid ARS and Correlated RS that have same properties of ARS but a bit enhanced to provide more advantages: the HARS process reduces the calculation cost of the Discrete Fourier Transform and the CRS has better aliasing supression. Though we limited our study in this paper to the main mode (ARS) in order to prove their common properties.

Second, the jittered random sampling (JRS) where a jitter (error) is applied to a uniform sampling grid, is a type of sampling that appears frequently in practical sampling systems because of uncertainty of sampling clocks due to hardware imperfections [9]. The sampling model in this case can be described by:

$$t_n = nT_s + u_n \quad T > 0 \quad n = 0,1,2, \ldots. \tag{4}$$

Where T_S is the mean inter-sample interval and u_n are iid variables with probability density function (PDF) equal to p(u) having variance σ^2 and a zero mean.

2. Condition of Random Sampling

2.1. Point Process Stationarity

As the sampling instants {t_n } are considered as points on the real timeline, they can be treated as a point process. Bilinksis and Mikelson defined the stationarity point process SPP as the probability of a sample occurring is the same everywhere on the time axis. If p(t) is the sum of all the individual PDFs: $p_n(t)$ which is related to the duration of consecutive inter-sample intervals, the SPP of a random point process is verified when:

$$p(t) = \sum_{n=1}^{\infty} p_n(t) = \frac{1}{\mu} \tag{5}$$

Where μ is the expectation of τ_k: μ=E[τ_k].

According to the Alias-Free theorem, if the random sampling sequence verifies the SPP condition of Bilinskis and Mikelson, the spectrum of the sampled signal is free of aliases [4, 7].

2.2.Time condition

The most usable distributions in sampling are: Gaussian and Uniform [3, 10]. In [4] a statistic parameter is introduced in order to measure the validity of the distribution to be used in sampling signals in reality. As mentioned before, the sampling instants $\{t_n\}$ are considered to form a random process, that should be simple. Thus, all t_k should be taken in increasing order:

$$0 \leq t_0 < t_1 < t_2 < \cdots < t_k < \cdots < t_n \, with \, \lim_{n\to\infty} t_n = +\infty$$

$$\text{With } kT_s - \frac{T_s}{2} \leq t_k < kT_s + \frac{T_s}{2} \; for \; 1 \leq k \leq n \qquad (6)$$

To verify this obvious condition, the random variable should be limited to [-0.5T; +0.5T] in JRS mode, and to [0.5T; 1.5T] in ARS mode. Consequently, the author in [4] concluded with a maximal limit for the statistic parameter σ/T_s equal to 0.2887.

2.3.ARS Stationarity

As, the PDF of the sum of two random variable is the convolution of their PDFs, the PDF of the k^{th} instant t_k is given by:

$$p_k(t) = \circledast_{i=1}^{k} p_i(t) \qquad (7)$$

This is the convolution repeated k times to obtain $p_k(t)$. $p_1(t)$ is exactly equal to $p_1(\tau)$. In [4], using the Central Limit Theorem, it is deduced that the PDF of t_k can be considered as a Gaussian distribution having mean=km and standard variation=$\sqrt{k\sigma^2}$ (when k→ ∞). Where τ_k has a pdf with mean=m and variance=σ^2:

$$p_k(t) = \frac{1}{\sqrt{2\pi k\sigma^2}} e^{\frac{-(t-kTs)^2}{2k\sigma^2}} \qquad (8)$$

In the same reference [4], in order to compute p(t) =Σ (p_k(t)), the author used Fourier transform of the sum of geometric series and the final value theorem to prove that :

$$\lim_{n\to\infty} p(t) = \frac{1}{T_s} \qquad (9)$$

Thus, the ARS mode is satisfying the SPP condition without determining any detail concerning the probability distribution.

2.4.JRS Stationarity

The probability density function (PDF) of the k^{th} instant is:

$$p_k(t) = p_1(t - kT_s) \; for \; 2 \leq k \leq n \qquad (10)$$

All the densities are similar but translated in time with a period equal T_S. The deduced form of the PDF function in JRS mode cannot give information concerning the stationarity, it depends on the chosen distribution: uniform or Gaussian.

For uniform distribution the PDF is:

$$p_k(t) = \frac{1}{T_s} \; with - k\frac{T_s}{2} \leq t \leq k\frac{T_s}{2} \qquad (11)$$

The pdf of the process t_n can be easily concluded:

$$p(t) = \sum_k p_k(t) = \sum_k p(t - kT_s) = const. \quad (12)$$

The formula (12) verifies the condition of point process stationarity declared already, the only remaining condition is to have a statistic parameter which verifies the temporal condition: σ/Ts<=0.2887 [4]. According to [5], a JRS process with Gaussian distribution can reach the point process stationarity after a certain delay (T_{ds}) depending on the distribution characteristics. In [4], it was proved that the stationarity is verified for σ /Ts=0.5>0.2887. Therefore, JRS is not recommended to be used with Gaussian distribution.

3. Time Quantization

Due to practical limitations and material implementation constraints, the real time axis couldn't be considered continuous. Therefore, time quantization should be applied on each recommended random sampling process to have a real exploration of such processes. Let Δ be the smallest time spacing step (that is generally declared by Analogue to Digital Converter (ADC) manufacturer or chosen by the user according to the system requirements). If T_S is the mean sampling period, q_T is then the temporal quantization factor:

$$\Delta = \frac{T_S}{q_T} \quad (13)$$

Let δ_{tk} be the distance between two consecutive sampling instants t_k and $_{tk-1}$:

$$\delta_{tk,q} = n\Delta, \text{ if } (n-1)\Delta < \delta t_k \leq n\Delta \quad (14)$$

The relation in (14) is conceived to model the behavior of sampling in digital applications. For example in our study we applied the Discrete Random Sampling using Arduino microcontroller. In such device, the sampling will be done by the ADC who is available each time step Δ. As the time between two samples is random, probably it will not be a multiple of Δ, so when the order of acquiring a sample is generated at the corresponding random instant the ADC will not execute the sampling until the next time step. To clarify the concept of time quantization process, Figure 1 is presented for the case where q_T=3.

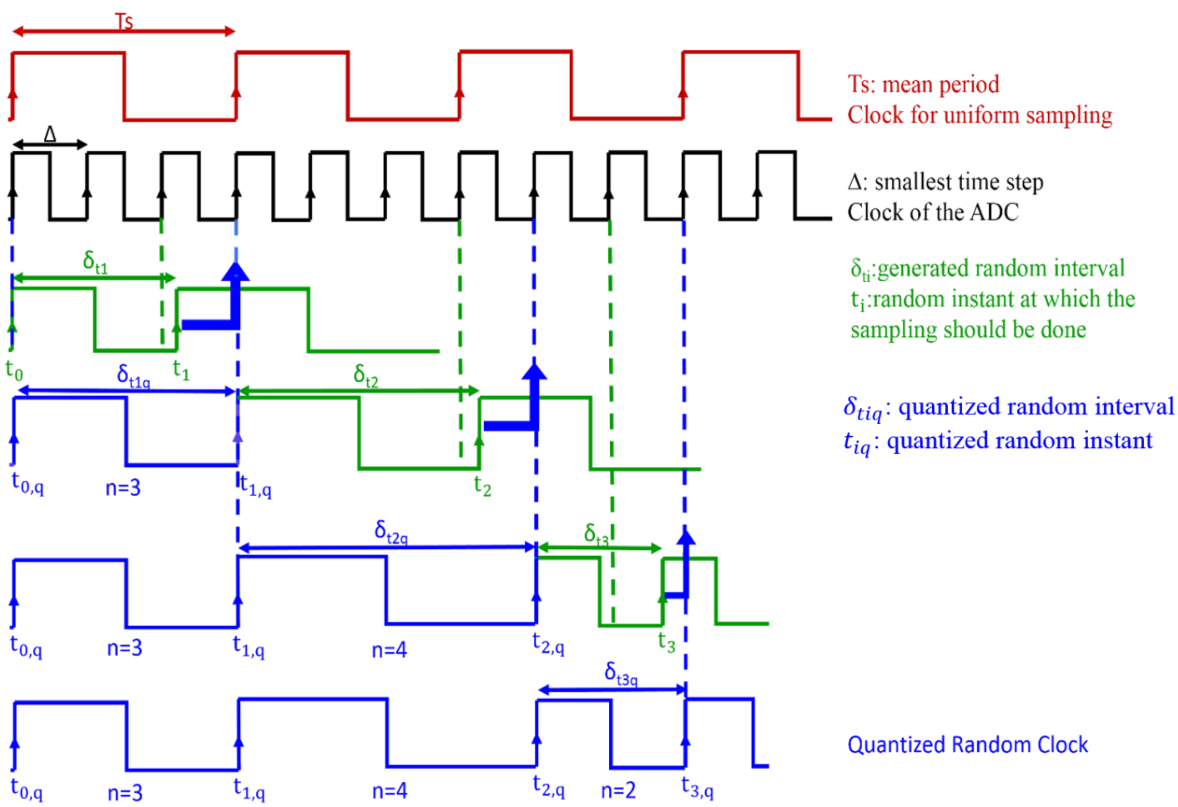

Figure 1: an example of time quantization mode for q_T=3

In Figure 1 a short example is figured to clarify how the quantization is done within random sampling. In the ADC, the acquisition of the samples takes place only on the ascending edges of its clock, whose period is Δ. When the random duration δ_{t1} is finished, the ADC is asked to acquire a sample, but at instant t_1 the ADC is not available yet, so it will wait until the next ascending edge or the next period Δ to execute sampling, so the random duration is quantized to be δ_{t1q} and the same for the instant $t_{1,q}$. The process is repeated for each sample, to obtain at the end a quantized random clock at which the discrete random sampling is done.

To study the effect of quantization on sampled signal, we go back to [3, 4] to deduce the characteristic function CF of the Time Quantized-RS ($\Phi_{1,q}(f)$):

$$\Phi_{1,q}(f) = \sum_{n=0}^{m-1} \Phi_1\left(f - \frac{n}{\Delta}\right) sinc(f\Delta - n) \tag{15}$$

Where Φ_1 is the CF of the distribution of random instants. It is clear that discretization has introduced a periodicity to the sampling sequence, which will affect the spectrum of the sampled signal. According to [5], the CF of the quantized interval τ_q becomes periodic of 1/Δ, so is the Power Spectral Density (PSD). In the interval [-1/(2Δ);+1/(2Δ)] no alias caused by the time quantization will occur.

A simple example is done by simulation on a sine wave having two frequencies: 100 and 150 Hz, to prove the limitation introduced by Δ in having alias in randomly sampled signals.

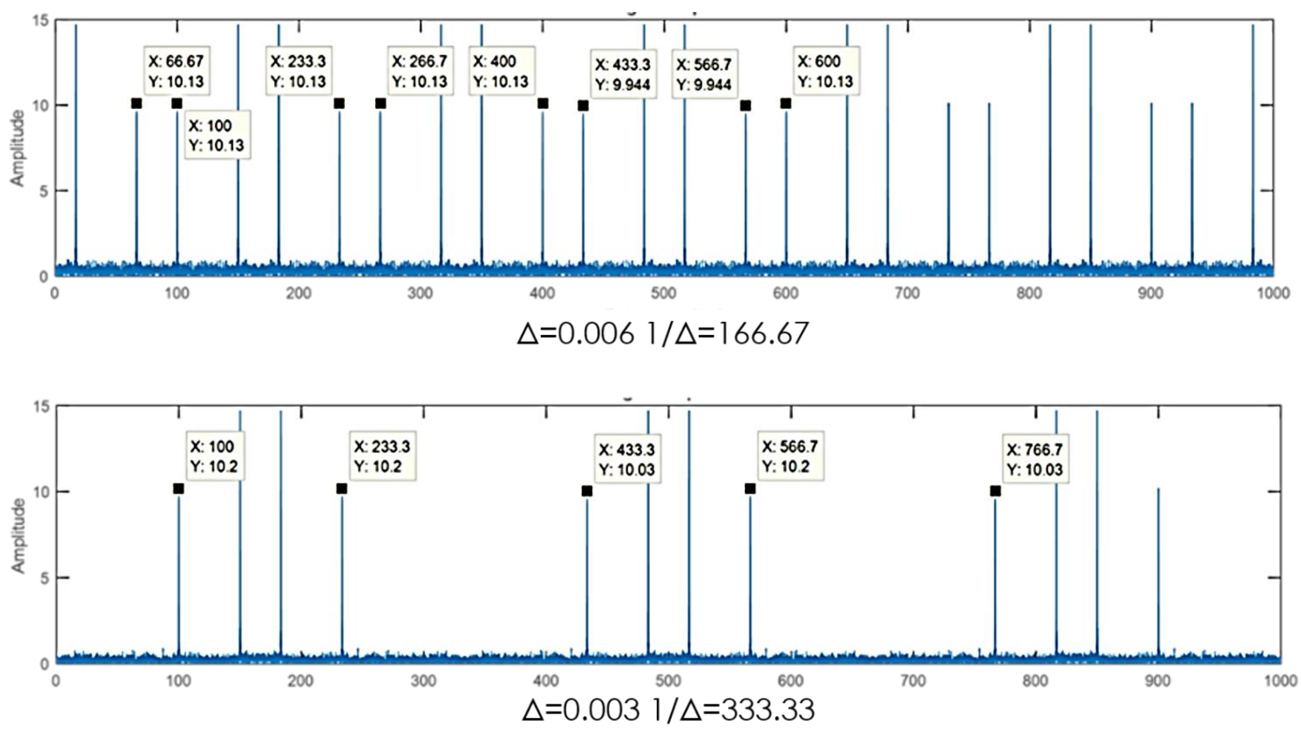

Figure 2: Two simulations of a signal having two frequencies 100 Hz and 150Hz with applying a quantization step = 0.006 in the top and step=0.003 in the bottom.

Like in uniform sampling, where the spectrum is repeated at each multiple of the sampling frequency, in quantized or discrete RS the spectrum of the signal is repeated every 1/Δ. In the bottom of figure 2, as the frequencies of the signal are 100 and 150 Hz, and the quantization step is 0.003 (1/Δ=333.3Hz), we see the repetition of 100Hz at 433.3Hz (333.3+100) and at 766.7 Hz (2*333.3+100) and the repetition of 150Hz at 483.3Hz (333.3+150) and at 816.7 Hz (2*333.3+150). The other alias are the opposite frequency (negative frequencies of the sine) of the replicas according to the reference, for example, 233.3Hz is the opposite of 433.3Hz according to 333.3 (=1/Δ) :233.3=-100+333.3, thus the whole replied spectrum is found within the interval [-1/(2Δ);+1/(2Δ)] that is centered at 1/Δ, which means [1/(2Δ);3/(2Δ)]=[166.7;500]. The same concept is shown for Δ=0.006 with the only difference that distance between replicas is shorter.

The relation between the sampling frequency and aliasing noise power is given by:

$$\frac{1}{\pi}\int_0^{\pi}\Phi_n(e^{jw})dw+\frac{\Delta}{E[\tau_k^q]}=1 \tag{16}$$

Where $\frac{1}{\pi}\int_0^{\pi}\Phi_n(e^{jw})dw$ is the average aliasing noise power of the sampled signal $x_s(t)$,that is reduced when the normalized average sampling frequency $\frac{\Delta}{E[\tau_k^q]}$ is increased $E[\boldsymbol{\tau}^q{}_k]$ is the mean value of the quantized random interval. So the author in [5] deduced that: once the average sampling frequency is fixed, the average aliasing noise power is fixed too, but according to the used probability distribution and the sampling process mode, the shaping of the aliasing noise power can be changed. For example, the uniform distribution used with ARS, when increasing the number of samples, the average aliasing power is reduced. The same distribution with JRS, when increasing the number of samples, the frequency range that is free of aliasing frequencies is increased.

4. Spectral Analysis of Randomly Sampled Signals

In Non-Uniform sampling many spectral analysis methods were suggested, some uses signal reconstruction, especially when it is a problem of missing or unreached data, these methods are based first on interpolation, slotted resampling or continuous time models, and then completed with uniform sampling spectral methods [12]. On the other hand, in compressive sensing methods based on exact spectrum reconstruction are used to go back with the inverse Fourier transform into time domain in order to reconstruct the whole signal, the most used are Matching Pursuit and Basis Pursuit where the reconstruction method is based on either l_0, l_1 or l_2 minimization [11]. In some other applications of NUS, as the Random Sampling, the purpose of spectral analysis is not the signal reconstruction, as the RS is chosen and not imposed. In fact, some methods for analyzing spectrum of randomly sampled signal are based on least square fitting like: Lomb and Scargle periodogram, Real-valued Iterative Adaptive Approach [12] and Fast Orthogonal Search [14], without the intention to do the signal reconstruction in time domain.

In our study we begin our analysis of the randomly sampled signal with the basic transformation from time to frequency domain: the Discrete Fourier Transform (DFT), by calculating the frequency component at the randomly chosen instants.

$$\widehat{X_s}(f)=\frac{1}{N}\sum_{k=0}^{N-1}x_k e^{-i2\pi f t_k} \tag{17}$$

Where N is the number of samples, x_k are the chosen samples at the instants t_k that are generated randomly. In non-uniform sampling, calculating the DFT at random instants is known as Point Rule NUT-DFT [13]. Theoretically, the DFT is a good method to estimate the Fourier transform of the signal in the random sampling context, but as the number of samples must be increased to enhance the quality of acquisition, the calculation cost becomes a real matter as the complexity of the algorithm is O (N^2). In our simulation study, we used basically the DFT to evaluate the random sampling effect on the sampled signal. In real signal acquisition it was a must to find a faster method to estimate the spectrum of long acquired signal. In [4 and 5] fast practical ways are proposed to calculate the spectrum of randomly sampled signal based on the use of the FFT: replace skipped samples by zeros (zero insertion) at the smallest time step Δ and calculate the FFT of the signal having the sampling frequency 1/Δ. In [4] the noise introduced by the zero insertion is minimized by averaging, while in [5] the spectrum is enhanced by the least square fitting. As the method of [4] is faster and easier to implement, we used it in processing the real signals that are sampled randomly, but instead of a simple averaging, we used the method of welch [18] to calculate the averaged spectrum of the data.

5. Simulation Study

The purpose of this study is to evaluate the random sampling and explore its impact on the sampled signal and consequently learn how to choose the sampling parameters in real applications. The simulated signal is a sine wave having 4 frequencies with different amplitude for each, a zero mean Gaussian noise, or white noise, 'wn' is added to the signal having a variance equal to 5:

$$S= 10*\sin (2*\Pi*1000*t) + 3*\sin (2* \Pi* 1100*t) + 5*\sin (2* \Pi* 1500*t) + 7*\sin (2* \Pi* 1510*t) + wn \qquad (18)$$

The time vector for sampling is generated by two methods: ARS and JRS. The intervals in ARS are random variables that follow a uniform distribution in the first case, and a Gaussian distribution in the second case. JRS is studied with the uniform distribution.

First, in ARS mode, time vector is simulated by adding random variables $\boldsymbol{\tau}_n$ that follow either a Uniform or a Gaussian distribution, as declared in (3). In case of uniform probability, the lower and upper endpoints of the interval [a;b] must be given, the mean μ of the distribution is equal to the mean sampling period and is given in term of a and b by (19):

$$\mu = T_s = \frac{a+b}{2} \qquad (19)$$

The variance σ^2 is also expressed in term of a and b to give the value of the standard deviation. To simplify the notation, we use the difference between the upper and the lower endpoints (that is equal to the interval length) instead of the standard deviation, we will call it deviation "D":

$$D = b - a \qquad (20)$$

To have a mean sampling period T_S and a maximum deviation (D), the interval must be: [$0.5T_S$; $1.5T_S$] as mentioned in section 2.2 so the maximum value of the ratio $\frac{D}{T_S}$ is 1, instead of 0.2887 as mentioned in section 2.2, let R be this ratio:

$$R = \frac{D}{T_S} \qquad (21)$$

Thus, if we have the value of T_S and R we can calculate the value of a and b to generate the needed time vector according to (22):

$$a = T_S\left(1 - \frac{R}{2}\right) \quad and \quad b = T_S\left(1 + \frac{R}{2}\right) \qquad (22)$$

Consequently, by varying the interval endpoints of the uniform distribution we can obtain different values of T_S and R.

In ARS with Gaussian distribution, the parameters that should be given are the mean μ and the standard deviation σ. As it is the ARS mode than the mean μ is equal to the mean sampling period T_S. So we can directly vary the mean to have different mean sampling periods. In addition, 99.7% of Possible values of a random variable that follows a Gaussian distribution are extended on the interval of [μ-3σ;μ+3σ] [15], so the length of this interval, that we already call Deviation is equal to 6σ. So when the T_S and R (= D/ T_S) are determined, the parameters of the Gaussian distribution are directly deduced in (23):

$$\mu = T_s \; and \; \sigma = T_s \frac{R}{6} \qquad (23)$$

In JRS mode, the random jitter u_n must follow a distribution with a zero mean, as declared in (4), so the mean sampling period is determined, independently from the distribution, and added to the generated clock or time vector. As the only distribution used with this mode is the uniform, the interval should be zero centered and the endpoints are opposites. The values of the endpoints are chosen according to the needed deviation D or the ratio R. In fact also in JRS, the ratio σ/ T_S is replaced by R and its maximum value (0.2887) is replaced by 1.

$$b = -a = R\frac{T_s}{2} \tag{24}$$

Thus, after generating the random jitter u_n, it will be added to the corresponding multiple of the chosen mean period (nT_s) to form the instant at which the random sample will be chosen.

After determining the mode and the distribution with its parameters, we must determine the number of samples N or the length of the sampled signal. We tried multiple signal length to analyze the effect of the samples' number on the resulted randomly sampled signal. In Table 1 all the tested values of T_S, R and N are depicted. But, to have a clear view of the impact of each parameter, for every value of T_S we changed the deviation or the ratio R and the number of samples from their minimum to their maximum.

T_S(sec)	0.002	0.005	0.0067	0.01	0.02	0.05	0.1			
N(pts)	25	50	80	100	150	200	400			
R (D/T_S)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

Table 1: Different simulated values of Ts, N and R

To have the ability of comparing the different modes and different distributions, the same values of these parameters are repeated in every case: ARS with uniform, ARS with Gaussian and JRS with Uniform. The time quantization step is taken equal to 5 microsecond to focus on the influence of random sampling. To evaluate the sampled signals, each case of random sampling (same mode, same distribution and same parameters) is generated 50 times, each time the DFT of the signal is calculated and squared to obtain the periodogram of the generated signal, and then, the average of all the periodograms of the 50 signals is taken to represent the case studied (the mode, the distribution and the chosen parameters) and thus analyzed to explore the results

At the beginning, we tried to observe the impact of RS on the ability to separate the sampled signal from the noise. In fact, the noise in the randomly sampled signal is not only the additive part 'wn' that is added in (18), another part is introduced by the random sampling process, and it is proved to be cyclo-stationary of order 2 as mentioned in [9], as the sampled signal is periodic. In figures 3 to 5, the value of the smallest amplitude is compared to the highest value of the noise, to examine how much the noise is eliminable in each case of RS: ARS with Uniform distribution, ARS with Gaussian distribution and JRS with Uniform distribution, and explore the impact of varying the mean sampling frequency (or period) and the number of samples taken. In each figure, the color of the plot represent the result for a specified frequency, for the same frequency two lines are drawn: the continuous line represent the value of the smallest amplitude of the signal and the dashed line represent the highest value of noise. These curves (continuous and dashed) show the variation of corresponding amplitudes with the variation of the number of samples in the signal. In fact, the simulation is done for all the already mentioned mean sampling periods (or frequencies) but only the most significant results are shown, for mean frequency equal to 500 Hz, 200Hz, 150 Hz and 50 Hz. The ratio R in these tests is fixed and equals 1.

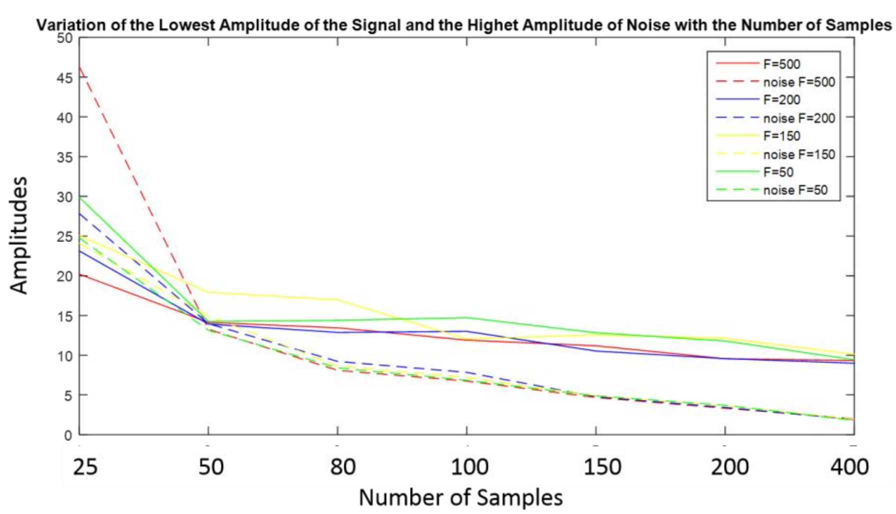

Figure 3: Comparison of the Smallest Amplitude of the Signal to the Noise in ARS with Uniform Distribution

We can obviously see that in ARS with Uniform, the signal is easily distinguished from noise for a number of samples greater than 50 for all the mean frequencies. The smallest amplitude of the signal is perfectly reconstructed and a way greater than the noise for a mean frequency of sampling that equals 50 Hz, a value that is much smaller than the Nyquist frequency.

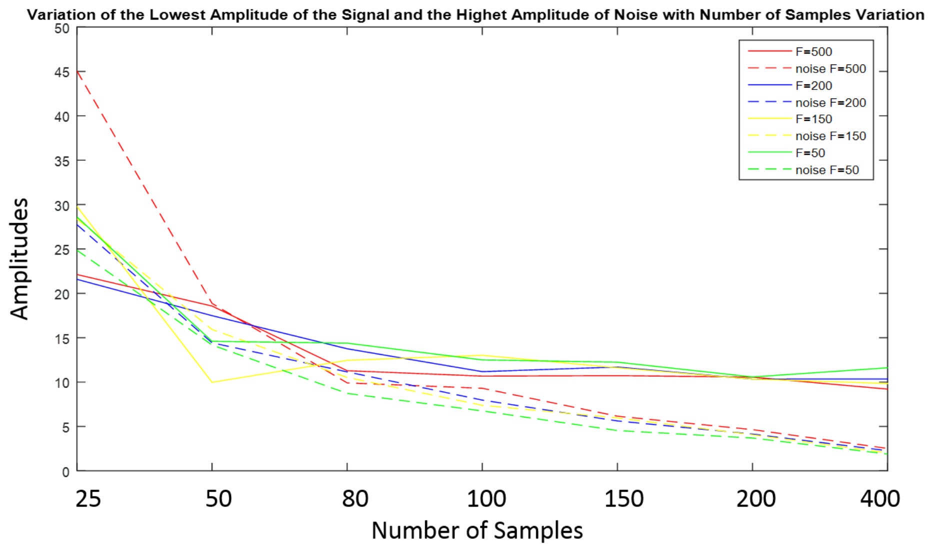

Figure 4: Comparison of the Smallest Amplitude of the Signal to the Noise in ARS with Gaussian Distribution

In ARS with Gaussian distribution the signal can be easily separated from noise for a number of points greater than 100, but we can see that also in this mode the mean sampling frequency can be a way smaller than Nyquist rate but with a limitation on number of samples that should be greater than 100.

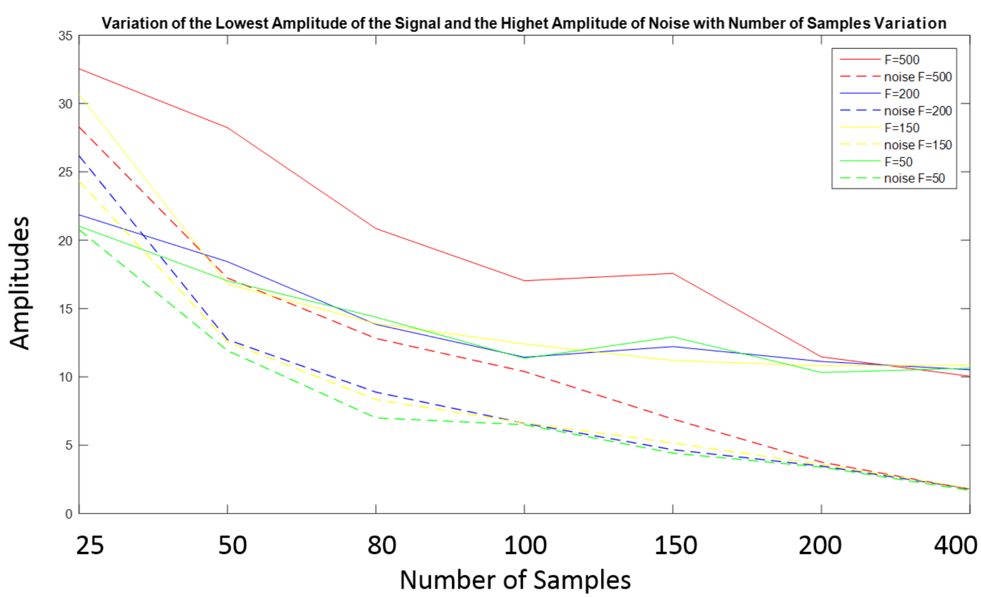

Figure 5: Comparison of the Smallest Amplitude of the Signal to the Noise in JRS with Uniform Distribution

In JRS with the uniform distribution, if we examine each signal amplitude (continuous line) with its corresponding noise maximum (dashed line having same color), we can deduce that for a number of points equal or greater than 50 the signal is easily separated from the noise. And as for other cases the number of samples, when increased, enhances the signal and reduces the noise to its minimum, for low and high mean sampling frequencies.

We can conclude from the comparison of these three ways of RS that the number of samples increases the performance and the low frequency can be easily used, but the ARS with Gaussian distribution provides results with good distinction of the signal from the noise, for a number of points greater than in ARS and JRS with uniform distribution. So, for cases where there is no limitation on data storage, the ARS with Gaussian distribution can be used. But, because of having such limitation, we preferred to continue our study with ARS and JRS with uniform distribution.

In addition, to observe the anti-aliasing property of random sampling, we examine the Amplitude/Alias (A/A) ratio with the variation of Deviation/T_S ratio already mentioned as R. In fact, R measures the "randomness" of the sampling process, so this analysis is done to study the effect of randomness on the anti-aliasing property. In each case, ARS and JRS with uniform distribution, we change the interval endpoints to have different R, the test is repeated for all frequencies, and in each case we have same results, so we will present the case of mean frequency that is equal to 10Hz. But as the number of points may influence on results, we will show the most significant cases. In Figures 6 and 7, the variation of A/A ratio with the variation of R is shown for number of points that is equal to 50, 80, 150 and 400, for both cases: ARS and JRS with uniform distribution.

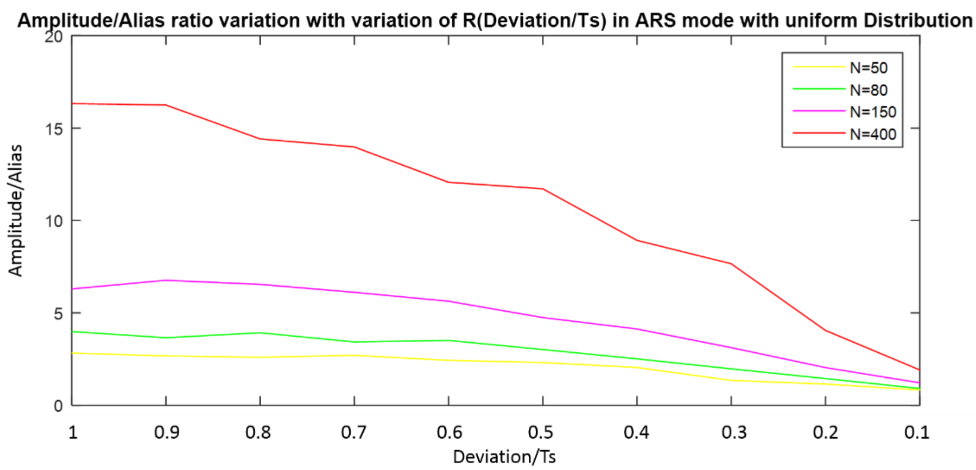

Figure 6: A/A variation with R variation in ARS with Uniform Distribution

To have a satisfying anti-aliasing property, the A/A ratio must be higher than 1. In Figure 6, we can see how this ratio decreases when R is decreased and how the number of points increases the A/A ratio but cannot avoid the impact of R diminution. However, in this mode (ARS with Uniform distribution) a value of R that is greater than 0.5 can provide a satisfying anti-aliasing property, which means that the length of the interval of the random variable must be at least half the mean sampling period.

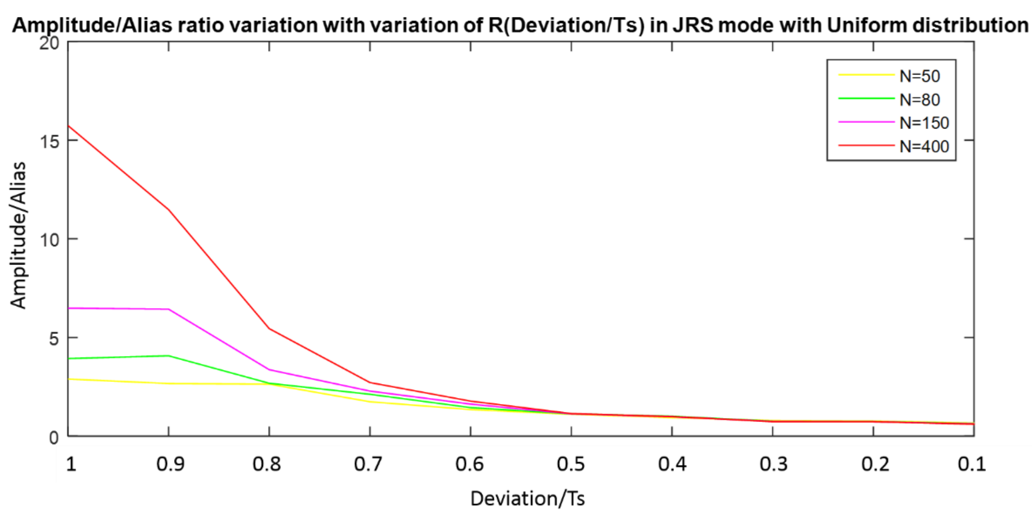

Figure 7: A/A variation with R variation in JRS with Uniform Distribution

Same conclusions concerning the impact of R and number of points on the A/A ratio can be said on the JRS with uniform distribution, but in this case, to have a good anti-aliasing, the ratio R must be greater than 0.8, which is a limitation for such sampling mode. Which means that when generating a uniform jitter, the interval length of the distribution must be greater than 80% of the value of the mean sampling period. Consequently, the JRS with uniform distribution has more limitations than the ARS with same distribution concerning the anti-aliasing property.

6. Hardware Implementation

6.1. Acquisition of Signals from Function Generator

In order to apply RS on real signals, a program was developed on Arduino Uno [19]. The main idea of the program is to generate random instants at which the samples are acquired by the ADC, both modes are applied: ARS and JRS with the uniform distribution. At the beginning, multiple tests were applied on a triangular waveform with a frequency of 1kHz and an amplitude of 2.5 V with an offset of 2.5V to be compatible with the ADC input (0 to 5V) , to reveal the impact of RS in practice. The ADC saves the value of the sample on 10 bits, the microcontroller clock is 16 MHz, with a possibility to be divided by a prescaler. So, the time quantization step Δ is determined by the ADC and by the clock of the microcontroller, when the clock has a low rate, large time granulation, the second and third harmonics were difficult to be detected due to the increased noise floor as mentioned in (16). Thus, we tried to keep using small Δ to focus on RS influence. To generate a random jitter or time interval that follows a uniform distribution we used the algorithm of Pseudo-Random Binary Sequence (PRBS). Many tests were done for different values of T_S, D and number of points. T_S must be declared in a register as a number of clock pulses, in ARS it should be the mean of randomly generated time intervals, in JRS it should be simply added to the randomly generated jitter. D is declared to limit the variation of the random variable (whether it is a jitter or time interval) to be compatible with time condition (6). Due to hardware limitations the ratio R couldn't have an exact value of 1, it is slightly greater than 0.8. The samples of the signal with their instants of sampling are saved, so the DFT of the randomly sampled signal can be calculated, in Matlab, as mentioned in (17). The offset of the signal is eliminated before calculating the DFT.

A uniformly sampled triangular waveform, with a frequency of 1 kHz, an amplitude equal to 2.5 and an offset of 2.5, is simulated on Matlab without noise. It has a spectrum with peaks on 1 kHz and its odd multiples as in Figure 8, and the fundamental has the highest amplitude while the other harmonics have reduced amplitudes (the offset is eliminated before calculating the FFT). To observe the two harmonics at 3 kHz and 5 kHz, the uniform sampling frequency is taken 10.2 kHz, the length of the signal is 2040 points.

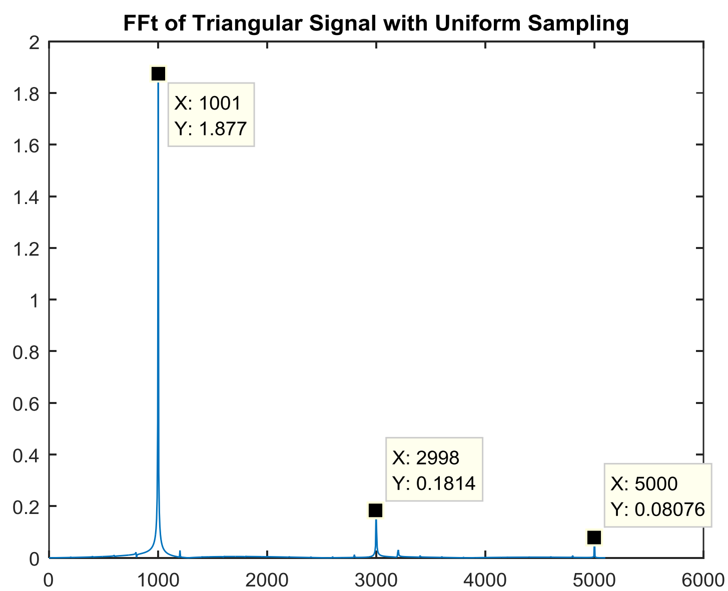

Figure 8: FFT of the Triangular Waveform with Uniform Sampling

The acquisition of the already mentioned triangular signal was done with different values of T_S, D and number of points, one of the most important results is shown in Figure 9, where we compare the DFT of the triangular signal sampled with ARS and JRS with the uniform distribution having same parameters. The mean frequency is approximately 200Hz, where the period is 4.99 ms and the ratio R is 0.8 (couldn't be greater due to hardware limitation). The number of samples taken is 4000 points. When comparing both results, we see that the aliases appears in the JRS mode, as the mean frequency of sampling is 200Hz , aliases appears around the fundamental with a distance of 200 Hz and its multiple, while in ARS no aliases appears. This aliasing is due to two reasons: the uniform distribution of the jitter is not perfect enough so the condition of stationarity is not satisfied as in (8), and the ratio R (deviation/Ts) is not greater than 80% as deduced from simulation. Moreover, the main fundamental and its first harmonic appears in both cases, the second harmonic has very low amplitude that can be easily covered by the noise due to hardware conditions and RS noise. Consequently, the ARS mode with uniform distribution is able to detect the peaks of the signals with a low frequency without aliasing with no limitations as in the JRS with Uniform distribution.

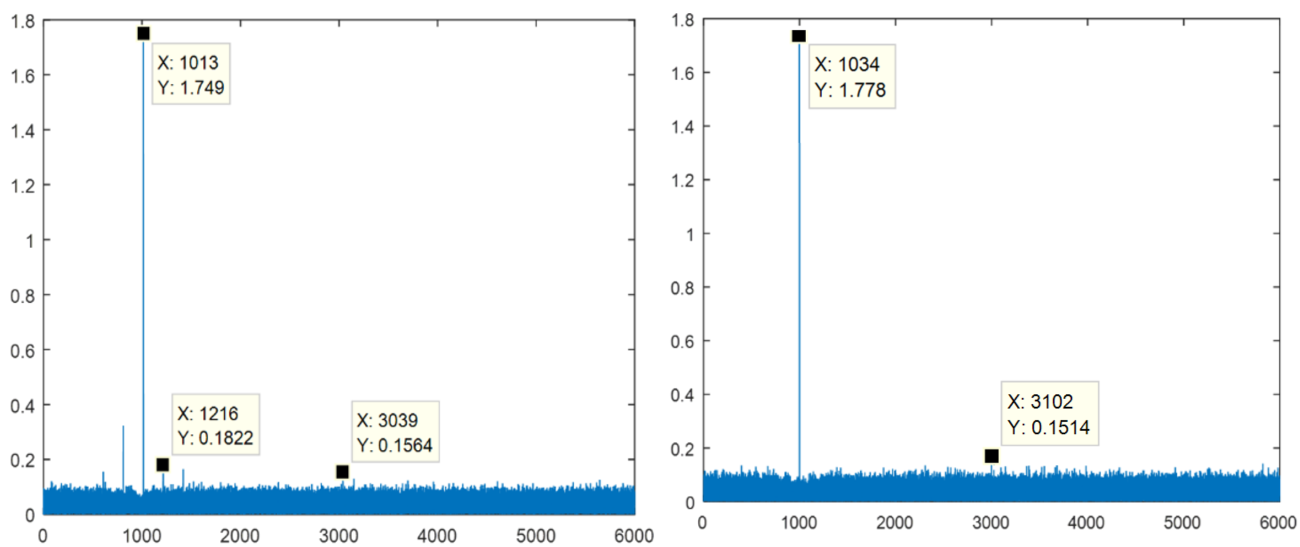

Figure 9: Comparison between two DFT of a triangular signal having a frequency of 1 kHz and sampled using a JRS (on the left) and an ARS (on the right)

6.2. Vibrational Signal Acquisition

The whole research on the random sampling and its different modes was done to study the possibility of its application on vibrational signals. After the test of random sampling on simple signal (triangular waveform), it was concluded that the ARS mode with the uniform distribution is the combination with less limitations to be applied on real signals, plus, the time granulation should be the smallest possible value, and the ratio R should be higher than 0.5. The frequency can be small enough and the number of points should be increased to have better result, in fact, these parameters are determined by the context of each application.

An experiment is done in the laboratory of LASPI (Laboratoire d'Analyse des Signaux et des Processus Industriels), on a rotating motor shown in Figure 10, having a bearing 6205 RS manufactured by MTM, with two possible cases: normal and defected inner race. The speed was 37 Hz. Vibrational signals are acquired by an accelerometer placed radially to the bearing as in figure 11. The output of the accelerometer is wired to the Arduino, so the signals can be sampled randomly.

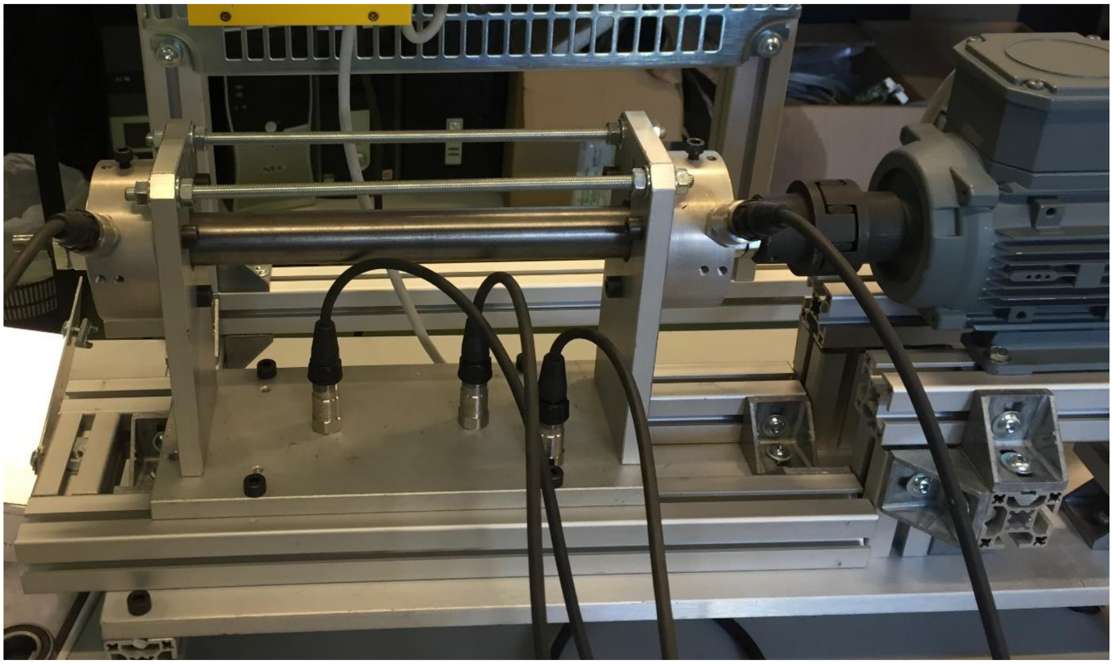

Figure 10: The Bench Containing the Rotating Motor with Bearing

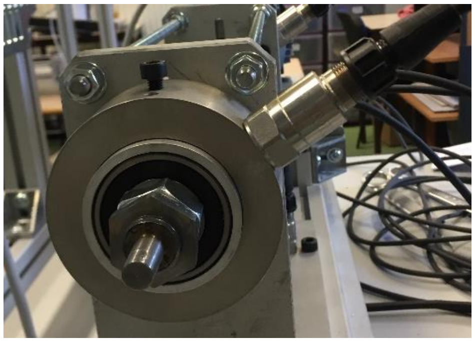

Figure 11: The Accelerometer Acquiring Vibrational Signals from the Bearing

Because of higher number of samples in real applications, the spectral analysis to be used in RS, couldn't be the DFT due to its high complexity. Thus, it was more advantageous to use the FFT after the zero Padding. In fact, the samples acquired by Arduino are saved with their random instants of sampling, and with the time granulation Δ which is the period of the clock of Arduino while the sampling is done. Consequently, the zero padding, or insertion, was done by adding zeros at each step Δ between the randomly chosen samples. So the acquired signal became longer and with a constant time step between samples, the samples are now the randomly acquired samples and the added zeros. As the time between samples is constant, the FFT can be applied on the acquired signals. The sampling frequency F_{0S} is given by (25):

$$F_{0S} = \frac{1}{\Delta} \tag{25}$$

In [4] to eliminate the noise added by the zero insertion the spectrum of the signal is averaged. In our study, we applied the Pwelch method in Matlab, to average the spectrum and obtain the periodogram of Welch, by dividing the signal in 8 segments with a Hamming window and an overlapping of 50%.

In first place, using National Instrument DAQ device, we acquire the vibration with uniform sampling frequency (51200 Khz) from the accelerometer (the same used in random sampling). The signal (of 100 000 pts), is processed in Matlab. In diagnostic, many tools of bearing signals' processing are based on the signal envelope's extraction to detect the important frequencies that give information about the bearing state: rotation frequency, defection characteristic frequency and others [17]. In this study, all the vibrational signals, randomly or uniformly sampled, are explored without any pre-processing or filtering.

First, we begin with the case of normal bearing, where the frequency to detect is the rotation frequency. In uniform sampling, only segments of 1000 points of the signal and its envelop are shown in Figure 12.

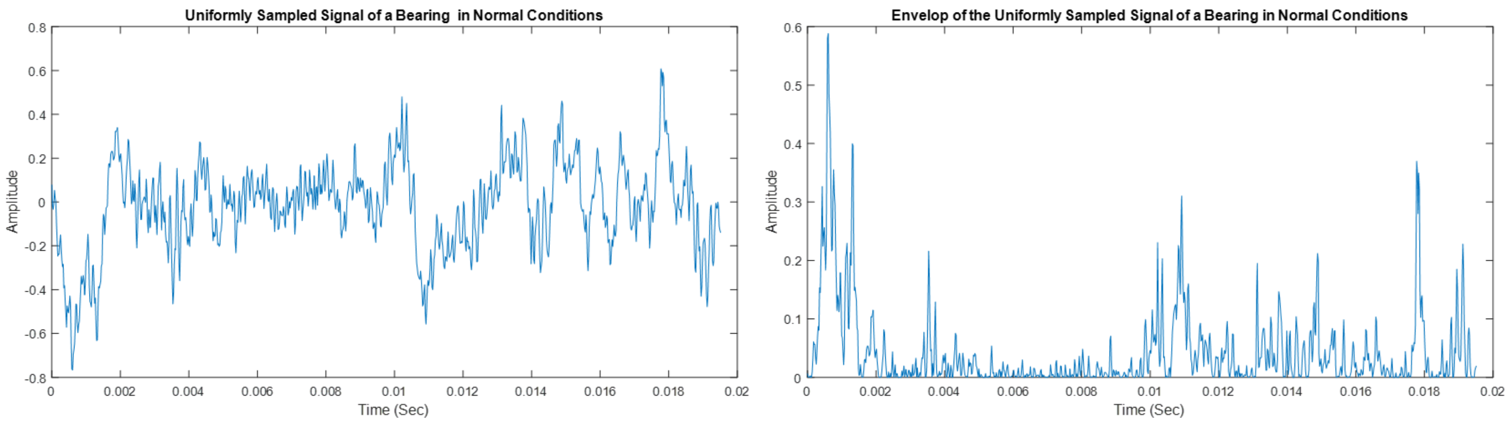

Figure 12: The Uniformly Sampled Signal of the Normal Bearing and its Envelop in Time Domain

To detect the frequency of rotation, the FFT of the envelop is calculated, and the PSD is presented to be compared to the PSD of the randomly sampled signal. Both are presented for the whole signal (100000 points) in figure 13.

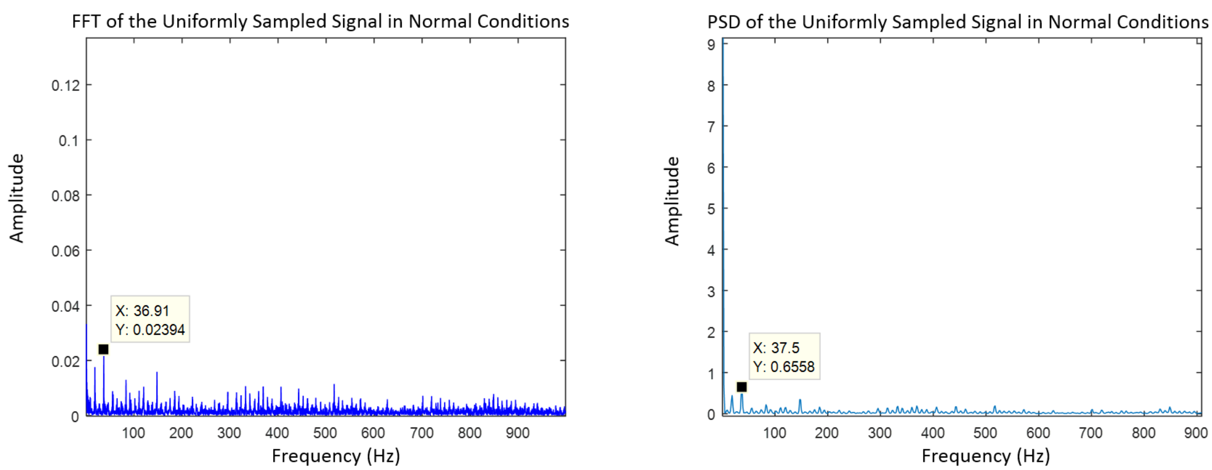

Figure 13: the FFT and the PSD of the Envelop of the Uniformly Sampled Signal of a Normal Bearing

It is obvious how in the FFT the rotation frequency is clearly detected with its harmonics. In random sampling, we choose the ARS mode with the uniform distribution to acquire vibrational signal, as it was deduced to have less limitations than other modes. As it is a low frequency signal, we chose the mean frequency 25 Hz, a ratio R equal to 0.8 and the number of acquired samples is 8000. The randomly sampled signal and its envelop are shown in Figure 14. Segments of 1000 points are presented in the figure. The FFT and the PSD of the envelop of the whole signal (8000 pts) are presented in Figure 15.

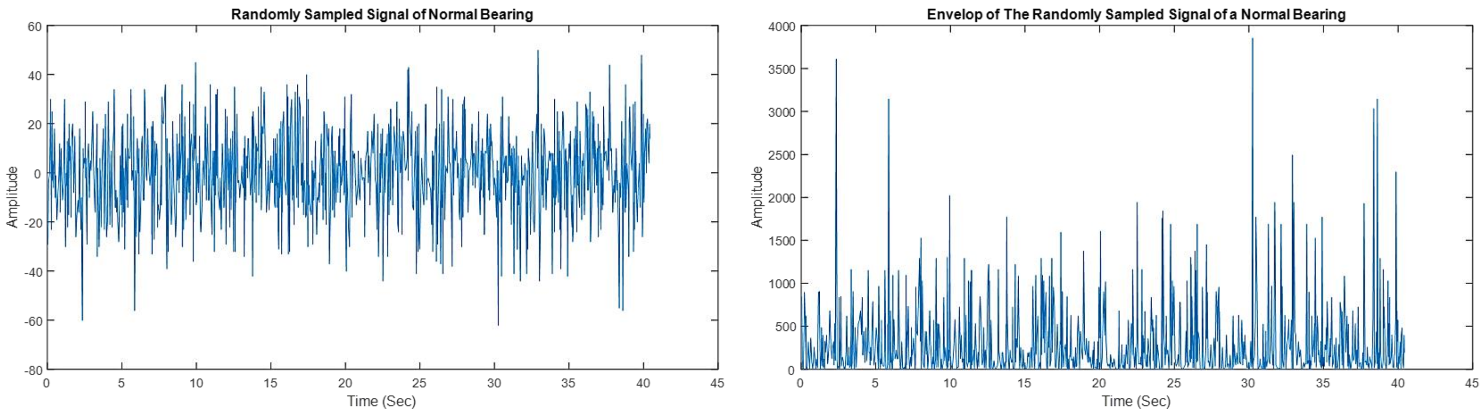

Figure 14: the Randomly Sampled Signal of the Normal Bearing and its Envelop

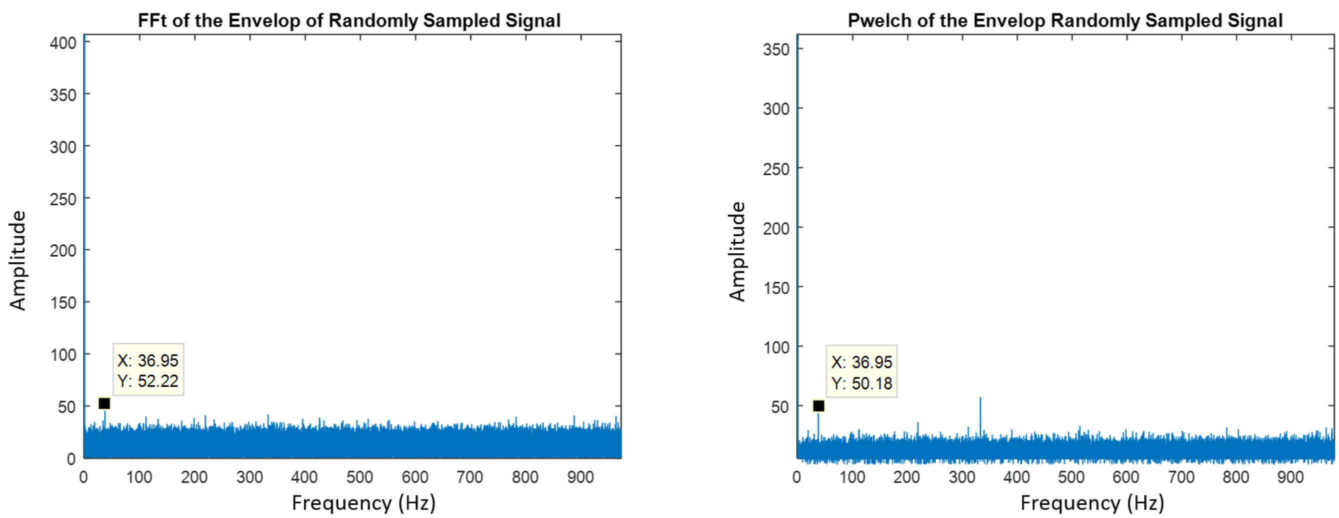

Figure 15: the FFT and the PSD of the Envelop of the Randomly Sampled Signal of the Normal Bearing

It can be deduced from Figure 15, that with a low sampling frequency (25 Hz) and a moderate number of samples (8000), the rotation frequency can be detected by using the ARS with uniform distribution. Though, the PSD is preferred, having better detection than FFT, due to noise elimination.

In case of defected bearing, the defection is situated in the inner race. According to the bearing type and its characteristics, the most important peaks are: 37 Hz and 199.4 Hz which is the defection frequency (inner race defection) [16]. As in the previous case, the uniformly sampled signal is presented with its spectrum, then the randomly sampled signal will be shown. The uniformly sampled signal and its envelop in time domain, are presented in Figure 16, segments of only 1000 points are shown. The frequency of sampling and the number of points are the same of the normal bearing case. The calculated FFT and PSD (power spectral density) of the envelop of the whole signal (8000 pts) are shown in Figure 17.

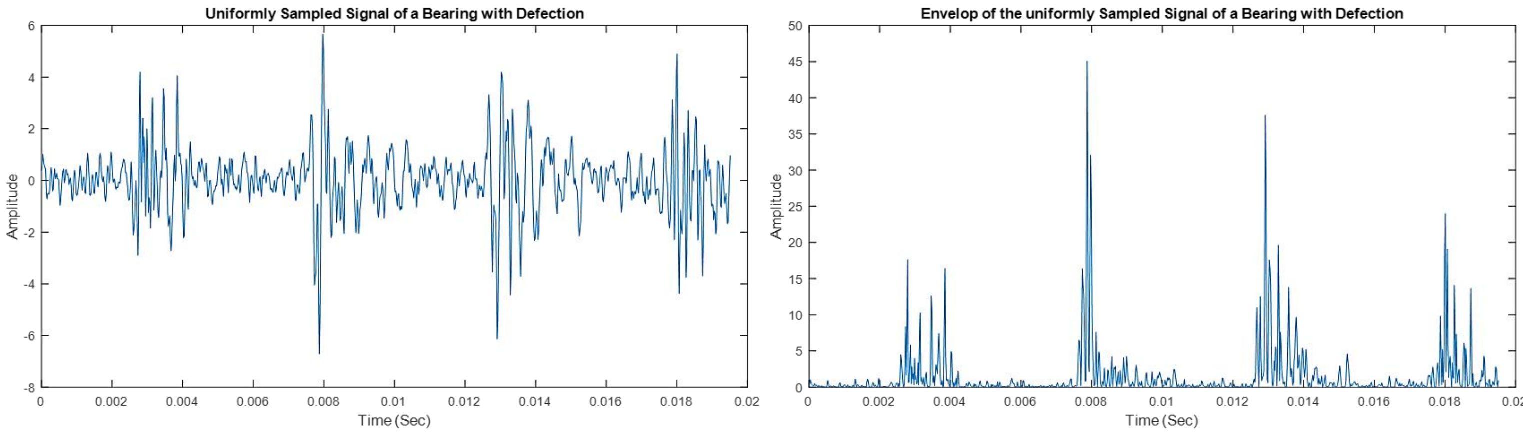

Figure 16: The Uniformly Sampled Signal of Defected Bearing Time Domain and its Envelop

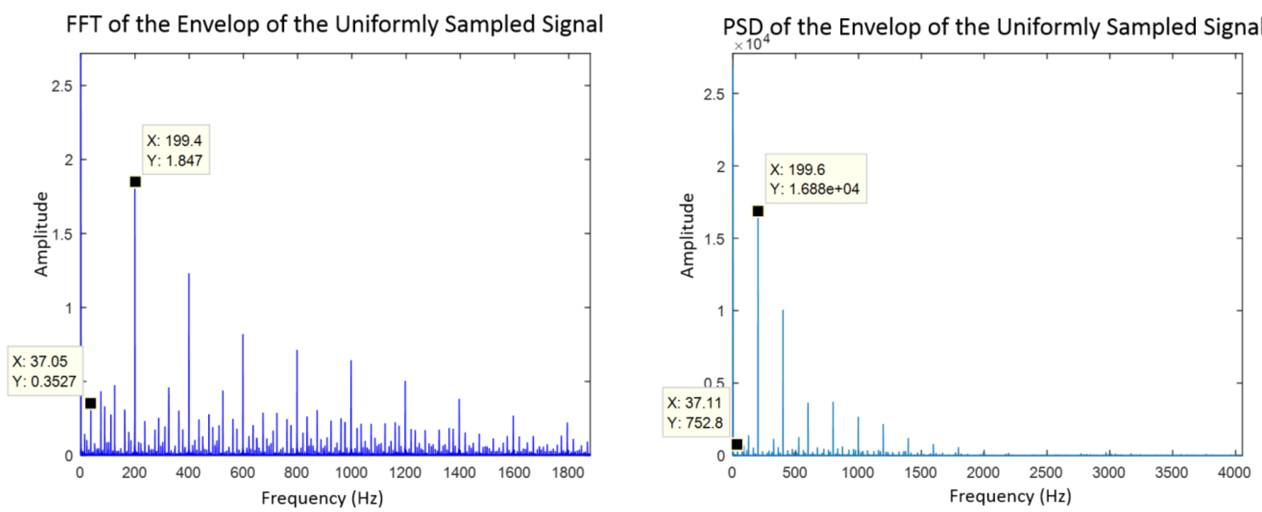

Figure 17: The FFT and the PSD of the Envelop of the Vibration of a Bearing Acquired with Uniform Sampling.

From figure 17, we can see how the rotation frequency and the defection frequency appear both in the FFT and in the PSD with their harmonics. The purpose of applying the RS on the vibrational signal is to obtain such results with lower frequency and moderate number of samples. So, for the same bearing in the same situation, we acquired the vibrational signal (8000 pts) from the same accelerometer with Arduino, using the ARS with the uniform distribution and a mean frequency 50Hz with a deviation that equals 80% of the mean period. The randomly sampled signal and its envelop in time domain are presented in figure 18. The FFT and the PSD of the envelop are presented in figure 19.

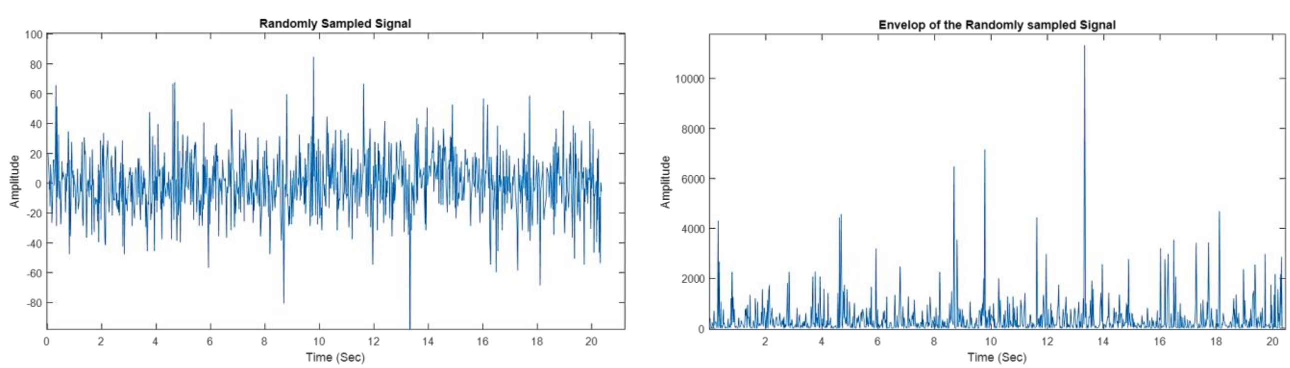

Figure 18: The Randomly Sampled Signal of the Defected Bearing and its Envelop in Time Domain

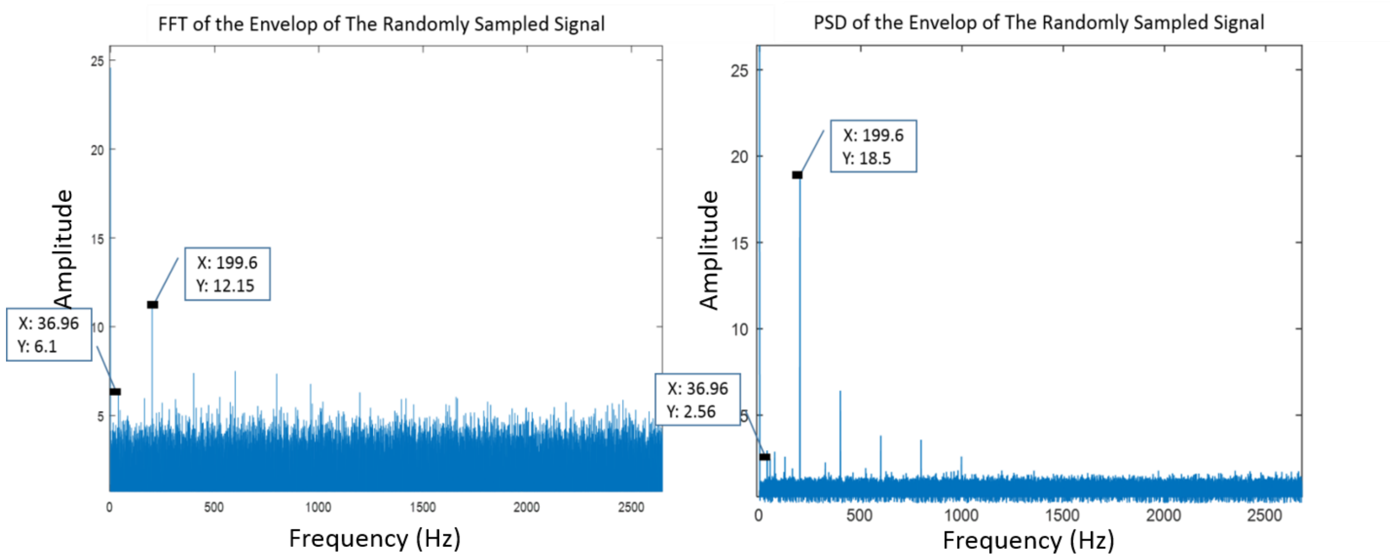

Figure 19: the FFT and the PSD of the Envelop of the Randomly Sampled Signal of the Defected Bearing using ARs with Uniform Distribution.

In ARS results, we can see that both peaks are detected with their harmonics, in the PSD the results are much better due to noise elimination. Compared to the randomly sampled signal of the normal bearing, the peaks of defection are very clear, which makes the diagnostic feasible with RS.

As for a first application of RS on vibrational signals this result is very satisfying when taking in consideration the frequency of sampling (50 Hz), the number of points (8000) and the unrequired anti-aliasing filter. Consequently, the random sampling seems to be a promising process to be applied on vibrational signals of rotating machinery so these could be easily monitored in real time applications, due to low sampling frequency, and thus be remotely supervised by using the IOT concept.

Conclusion

Finally, after a brief review on the Random sampling and its different modes, we conclude that the stationarity condition is approved by the ARS mode with any random distribution and by the JRS with only the uniform distribution. Though, the simulation study showed some limitations of the ARS with the Gaussian distribution and the JRS with the Uniform distribution that could be treated with an increased number of samples and high ratio of Deviation/T_S which need a high performance of hardware. So, we choose the ARS with uniform distribution to be used in practice. The spectral analysis of randomly samples signal is done by using the DFT for simulated signals and by the FFT and PSD (of welch) after zero insertion for real signals. A comparison between vibrational signals acquired from a normal and a defected bearing, using both uniform and random sampling is done. In RS, the resulting spectrums in both cases, normal and defected bearing, are clear and give the needed information to do the diagnostic. At the end, we figure out that the ARS is applicable in machine monitoring domain, offering a diagnosis with low frequency (a way smaller than Nyquist frequency) without demanding a large amount of data neither an anti-aliasing filter. Thus, the random sampling, inspired by the compressive sensing, can simplify the implementation of machine monitoring to be used in remote application having low frequency rate and thus easily managed in real time operations. Having such potential, random sampling should be studied in order to reveal all its specifications and limitations, especially the noise that is introduced by this process and need to be eliminated to enhance the performance of this promising way of sampling.

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