# Instantaneous angular speed estimation for multi-stage wind turbine gearboxes

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## Abstract

To solve the problem of non-stationary speed conditions in rotating machinery, order tracking has become a popular practice to synchronize the Fourier analysis to the shaft rotation. Being able to estimate the instantaneous angular speed (IAS) accurately, enables analysts to identify machine events linked to the shaft rotation much easier. The IAS essentially allows to resample the vibration signals in the angular domain instead of the time domain. Wind turbine gearboxes however are usually multi-stage gearboxes, consisting of multiple shafts, rotating at different speeds. Fitting a sensor (e.g. a tachometer) to every single stage is not always feasible. As such there's a need to estimate the IAS of every single shaft based on the vibration signals measured by the accelerometers. This paper investigates the multi-order probabilistic approach for IAS estimation as proposed by Leclère et al. [1]. This method takes into account the meshing orders of the gears present in the system. We investigate the possibility to apply the resampling procedure for a single shaft only. Furthermore we investigate the potential for improvement by applying a more sophisticated weighting of the different meshing orders in the estimation of the probability density functions. The least squares method is examined for this purpose. Finally, this technique is then validated on experimental data.

## 1 Introduction

Vibration analysis is a commonly used tool for condition monitoring and fault diagnosis of wind turbine drivetrains and it has proven to be one of the most effective tools in general for mechanical fault detection [2]. Unfortunately however, wind turbines are subject to highly varying conditions and thus the speed of the rotor input shaft is usually highly non-stationary. Vibration signals originating from the gearbox are consequentially angle-time cyclo-non-stationary signals [3]. Most processing tools based on frequency analysis of the signal are only applicable however for stationary speed regimes. Speed variation causes the spectral content to smear, which encumbers most conventional fault detection techniques. A possible solution for the non-stationary speed can be to measure the instantaneous angular speed (IAS) by use of an angle encoder or tachometer. The instrumentation of a system with such a device is however not always cost efficient or feasible. Therefore recent developments focus more on directly extracting the IAS from the vibration signal itself. Several approaches are already available and some commonly used methods include Vold-Kalman filtering [4, 5], Gabor and wavelet transforms [6, 7], band-pass filtering[8] and the Teager-Kaiser energy operator[9]. However, the majority of these techniques rely on the presence of a frequency band that consists of only a discrete harmonic of the rotation frequency with a high signal-to-noise ratio (SNR). Other signal components like harmonics that are not related to the rotation speed of interest or structural resonances, are not desirable in the frequency band and usually ruin the chances of successful IAS estimation. Since wind turbine gearboxes are usually a complex system consisting of multiple (planetary) stages, the vibration signals measured from such a complex system contain multiple resonances and asynchronous harmonics. As such, the assumption of a frequency band containing only one harmonic with high SNR cannot be guaranteed.

The limitation of only taking taking into account one harmonic suggests an approach that utilizes as many harmonics as possible in order to increase the amount of information and correspondingly the accuracy. This paper examines exactly such a method, namely the multi-order probabilistic approach (MOPA) as first proposed by Leclère et al. [1]. This method uses the available information about the system and does not require to associate one harmonic with the correct order. Knowledge of the gear ratios only is sufficient to extract from the vibration signal an accurate estimation of the IAS. This paper investigates the possibility to optimize the use of the available knowledge of the systems mechanics in the form of the order ratios.

# 2 Methodology

This paper investigates the possibility to improve the accuracy and/or the computational requirements by performing an a priori optimization of the selection of fundamental orders through least squares. This section briefly describes the theoretical background of the multi-order probabilistic approach (MOPA).

## 2.1 Multi-order probabilistic approach

The general idea behind the multi-order probabilistic approach (MOPA) as proposed by Leclère et al. [1] is based on regarding the instantaneous spectrum (which can be obtained through a short time Fourier transform) of the vibration signal as a probability density function (pdf) of the IAS $\Omega$. Consequently, if the spectrum has a high amplitude at frequency $f$, there is a high probability that the shaft frequency is equal to $f/H_i$ with $H_i$ being the excitation order or for the cases described below the gear ratios. It is important to define a range for the IAS in which the user expects the IAS to reside. This range has a lower bound $\Omega_{min}$ and an upper bound $\Omega_{max}$. The following pdf can then be constructed:

$$\begin{cases} [\Omega|H_i] = \frac{1}{\xi_i}A(H_i\omega) & \text{for } \Omega_{min} < \omega < \Omega_{max} \\ [\Omega|H_i] = 0 & \text{for } \omega < \Omega_{min} — \omega > \Omega_{max} \end{cases} \tag{1}$$

with $A(f)$ a whitened version of the vibration signal's spectrum and $\xi_i$ a normalization factor to make sure the pdf has unit area:

$$\xi_i = \int_{\Omega_{min}}^{\Omega_{max}} A(H_i\omega)d\omega. \tag{2}$$

The purpose of the whitening is essentially to reduce the influence of resonances on the generated pdf, since it is undesirable to give a too high probability to a certain part of the spectrum only due to the increased amplitudes because of a resonance. The used whitening technique should be chosen based on the application.

To improve the IAS estimation and utilize more of the information potential of the spectrum, one has to include more than just one pdf based on one gear ratio or meshing order. Afterwards these different pdfs can be combined together in one pdf by multiplication. Equation1 does not take into account the possibility that a part of the spectrum for a certain harmonic $H_i$ can exceed the Nyquist frequency. In this case the pdf is made uniform above $f_{max}/H_i$:

$$\begin{cases} [\Omega|H_i] = \frac{1}{\xi_i}A(H_i\omega) & \text{for } \Omega_{min} < \omega < f_{max}/H_i \\ [\Omega|H_i] = \frac{1}{\Omega_{max}-\Omega_{min}}[\Omega|H_i] = 0 & \text{for } \omega < \Omega_{min} — \omega > \Omega_{max} \end{cases} \tag{3}$$

with $\xi_i$ now:

$$\xi_i = \frac{\Omega_{max} - \Omega_{min}}{f_{max}/H_i - \Omega_{min}} \int_{\Omega_{min}}^{f_{max}/H_i} A(H_i\omega)d\omega. \tag{4}$$

The method's inputs are thus an approximate range for the IAS, the meshing orders and the vibration signal. For every order a pdf is then constructed based on the signal's instantaneous spectrum and rescaled to the given range for the IAS. Next, the pdfs are multiplied to combine the information of all the orders so that the main corresponding estimate for the IAS becomes the most dominant peak in the pdf.

Currently the pdfs are still independently generated for each time step and thus do not guarantee any continuity of the IAS, which is a logical assumption for any mechanical system. Due to the inertia of the rotating shafts, strong acceleration or deceleration is improbable. As such, to improve the results further, an a priori of continuity is introduced for the IAS. The concept relies on generating for each time step a pdf that is based on the pdfs of several time steps before and after the central pdf. Appropriate weighting of these pdfs is done by convolving the pdf with a centered Gaussian and the time relationship is introduced by letting the variance depend on the time between the considered pdf and the central pdf. The pdf at time step $j$ generated by the pdf at time step $j+k$ is defined as:

$$[\Omega_j]_{j+k} = \int_{\Omega_{min}}^{\Omega_{max}} [\Omega_j|\Omega_{j+k}]d\omega \propto \exp(\frac{\omega^2}{2\sigma_k^2}) * [\Omega_{j+k}] \tag{5}$$

with $[\Omega_j]_{j+k}$ the pdf at time j that can be obtained by convolution of the pdf at time $j+k$ $[\Omega_{j+k}]$ with a centered Gaussian, and $\sigma_k = |\gamma k \Delta_t|$ with $\Delta_t$ being the time step, $\gamma$ the standard acceleration of the IAS. Similar to the previous step in which the pdfs corresponding to the different orders have to be multiplied for each time step to obtain a single combined pdf, there are now again multiple pdfs for every time step $j+k$ belonging to time steps before and after time step $j$. Thus the final step is to multiply again all the pdfs for every time step:

$$[\Omega_j]_s \propto \prod_{k=-K}^{K} [\Omega_j]_{j+k}. \tag{6}$$

### 2.1.1 Least-squares optimization for order selection

The MOPA method, as described in Section2.1, does not take into account the possibility that some of the harmonics of the fundamental orders can have weak or even absent spectral signatures. This suggests a possible improvement of the technique through obtaining an optimized set of orders first. The proposed approach makes use of a reference speed signal to find an optimal weighting of the different orders. The main drawback is the requirement of a reference speed signal, but it only has to be measured once in the beginning of the measurements by use of a tachometer or an angle encoder. Additionally, most wind turbines in a farm use the same gearbox type requiring only the instrumentation on one turbine. The reference speed signal is then used to generate a two-dimensional matrix containing for every time step in the spectrogram a Gaussian probability density function. The mean of every Gaussian curve is the reference speed in Hertz for that time step and the standard deviation can be determined as the product of the standard acceleration $\gamma$ of the shaft and the length of the used time windows in the spectrogram. This representation forms the ideal situation where the pdf contains only one peak related to the shaft speed and no noise or unrelated peaks that could skew the expected value extraction of the pdf. An example visualization of this matrix can be seen in Fig.1.

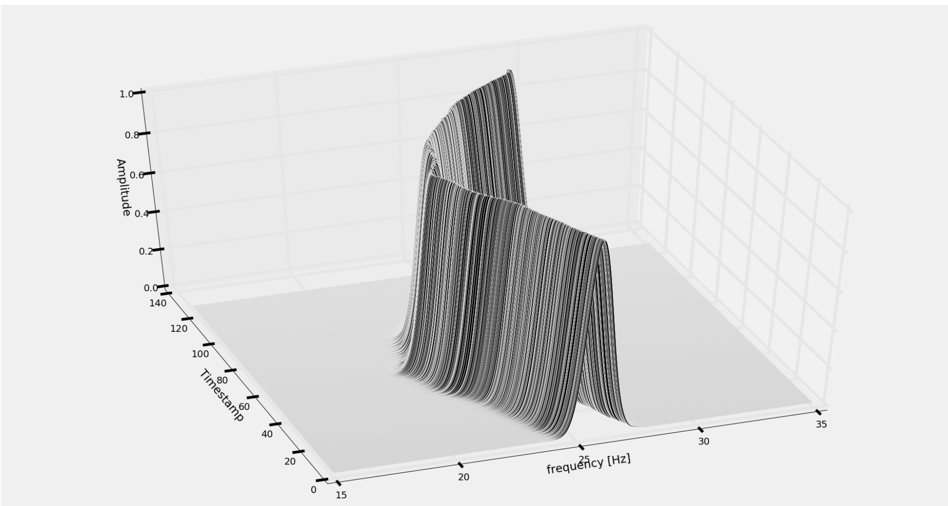


Figure 1: Visualization of the Gaussian probability density map used as a reference for the optimization.

Table 1: Fundamental orders related to high-speed shaft.

| Gear pair | Order value |
|---|---|
| 1 | 1 |
| 2/3,1/2 | 1.025459229 |
| 4/5 | 5.316666667 |
| 6/7 | 29 |
| 8/9 | 15.225 |
| 10/11 | 6.619565217 |

The least squares regression problem is then simply formulated as follows:

$$A_{t_i} x_{t_i} = b_{t_i} \tag{7}$$

with $b_{t_i}$ being the Gaussian pdf generated from the reference speed at time $t_i$, $A_{t_i}$ the matrix with in every column the pdf belonging to an harmonic order at time $t_i$ after logarithmic transformation of the pdf spectra, and $x_{t_i}$ the coefficients to be calculated. The pdfs are normalized to have unit area before the coefficients are calculated.

## 3 Experimental application on CMMNO 2014 contest data

The proposed optimization approach is investigated using a well-documented data set originating from a diagnosis contest held in light of the International Conference on Condition Monitoring of Machinery in Non-Stationary Operations (CMMNO) in 2014.

The vibration signal used for this analysis was measured on the gearbox housing of a wind turbine near the epicyclic gear train and sampled at 20 kHz. The goal was to estimate the IAS of the high-speed shaft (carrying gear #7 in Fig.2). This estimate was then compared with a reference speed signal measured by an angle encoder. The length of the measurement was approximately 550 seconds.

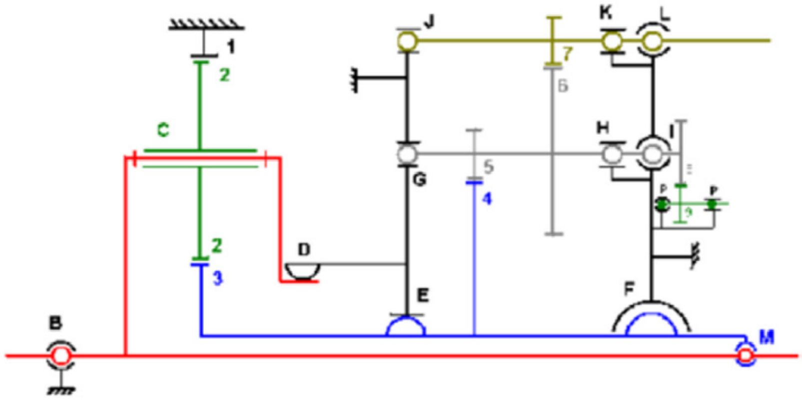


Figure 2: Visualization of the wind turbine gearbox used in the CMMNO 2014 diagnosis contest.

In the first step a spectrogram is generated using a rectangular window with a window length of 0.5 seconds, 1.5 seconds of zero padding, and an overlap of 0.25 seconds. Figure 3a displays the resulting spectrogram, in which there are some clear resonances (*red horizontal lines*) to be seen. Therefore the spectrogram is whitened to remove these resonances as is shown in Fig.3b.

The next step is the generation of the pdfs for every meshing order and combining them in order to obtain a single pdf representing the most likely frequency of the IAS. In total there are 6 fundamental orders that are related to the high-speed shaft, see Table 1.

Initially, ten harmonics of all fundamental orders will be considered to obtain matrix $A_{t_i}$, thus producing 60 harmonic orders. For every time step, the least-squares solution can be found and the resulting coefficients are tracked for every order in time. Figure 4a displays the coefficient values for the 60 orders for every time step.

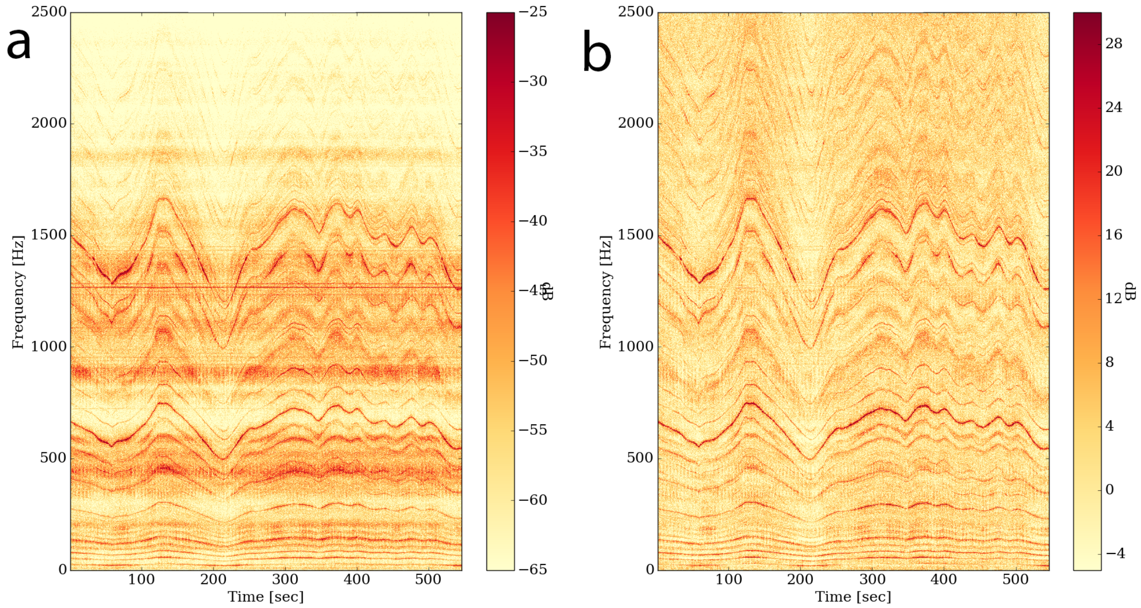


Figure 3: (a) Spectrogram of the CMMNO 2014 vibration signal of the wind turbine gearbox housing. (b) Whitened spectrogram

As can be seen, the least squares solution mostly suggests the same orders to be weighted more throughout time. Figure 4b shows the time averaged value of the coefficients for every order.

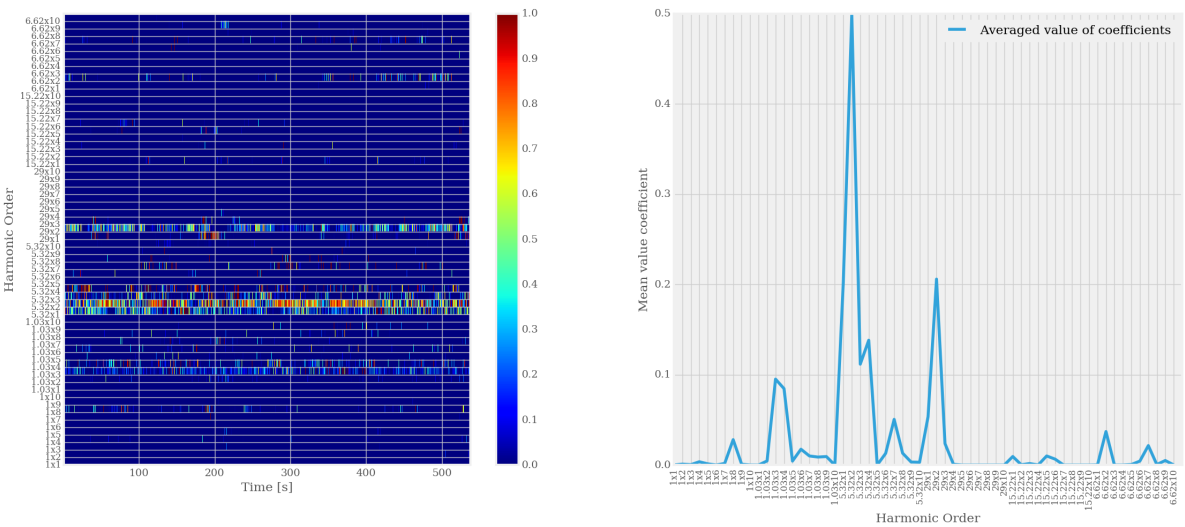


Figure 4: (a) 2D-map of the coefficient values for every harmonic order (from 1 to 10 for every fundamental order) and for every time step. (b) The time averaged coefficient values for every harmonic order.

Four different ways for using the found coefficients in the multi-order probabilistic approach are investigated:

1. Using all harmonic orders with equal weighting (standard approach)
2. Using all harmonic orders with the time averaged coefficients as weights.
3. Using only the dominant orders with equal weighting.
4. Using all harmonic orders with time dependent coefficients.

Table 2: Selected dominant harmonic orders.

| Fundamental order | Harmonic numbers |
|---|---|
| 1 | $8^{th}$ |
| 1.025459229 | $3^{rd}$,$4^{th}$,$6^{th}$,$7^{th}$ |
| 5.316666667 | $1^{st}$,$2^{nd}$,$3^{rd}$,$4^{th}$,$6^{th}$,$7^{th}$,$8^{th}$ |
| 29 | $1^{st}$,$2^{nd}$,$3^{rd}$ |
| 15.225 | $5^{th}$ |
| 6.619565217 | $2^{nd}$,$7^{th}$ |

Thus the pdfs are obtained for each one of the four described approaches. The threshold for the dominant order selection was set very low at 0.01 for the time-averaged coefficients and resulted in 18 orders to be taken into account for the pdf construction instead of 60. Table 2 gives an overview of the selected dominant orders.

The last step then is to introduce the continuity condition, which performs a smoothing of the pdfs. Now that the pdfs for all four cases are constructed, the expected value is extracted for every time step. This results in the instantaneous speed curves shown in Fig.5. As can be seen, there is very little difference between the estimated shaft speeds of the four approaches and the reference shaft speed.

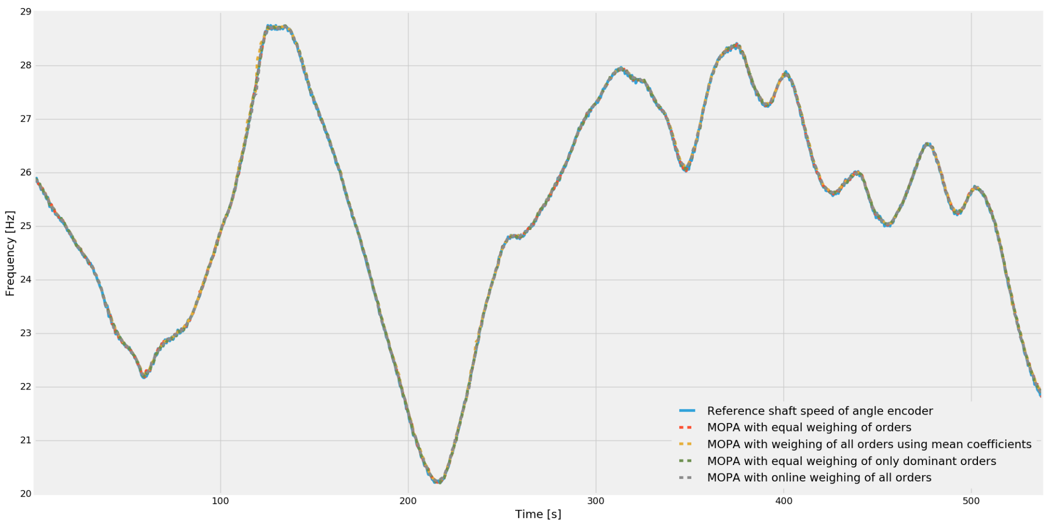


Figure 5: Shaft speed estimation results of the different weighting approaches for the MOPA method.

The relative errors and the root-mean-square-error between the estimated speeds and the reference is shown in Fig.6. All four methods perform acceptably, with the standard MOPA and the approach using only selected dominant orders performing the best (RMSE of respectively 0.0233Hz and 0.0248Hz). This result suggests the approach using only the selected orders with equal weighting since it's the least computationally expensive and still accurate. The standard MOPA took 154 seconds (with an i5-5300U CPU at 2.3GHz) and 9.34GB of RAM memory, while it took only 31 seconds and 1.27GB RAM using the reduced set of orders.

To corroborate the selection of the dominant harmonic orders, the mean order spectrum is calculated. This spectrum is obtained after transforming the spectrogram in an RPM-Frequency map using the estimated instantaneous speeds and taking the mean value for every order. Figure.7 shows the logarithmic mean order spectrum in blue and the selected dominant orders with red dashed vertical lines. These red lines generally correspond quite well with the peaks present in the spectrum, although some of the peaks would be less straightforward for a human to select due to smearing or lower amplitude.

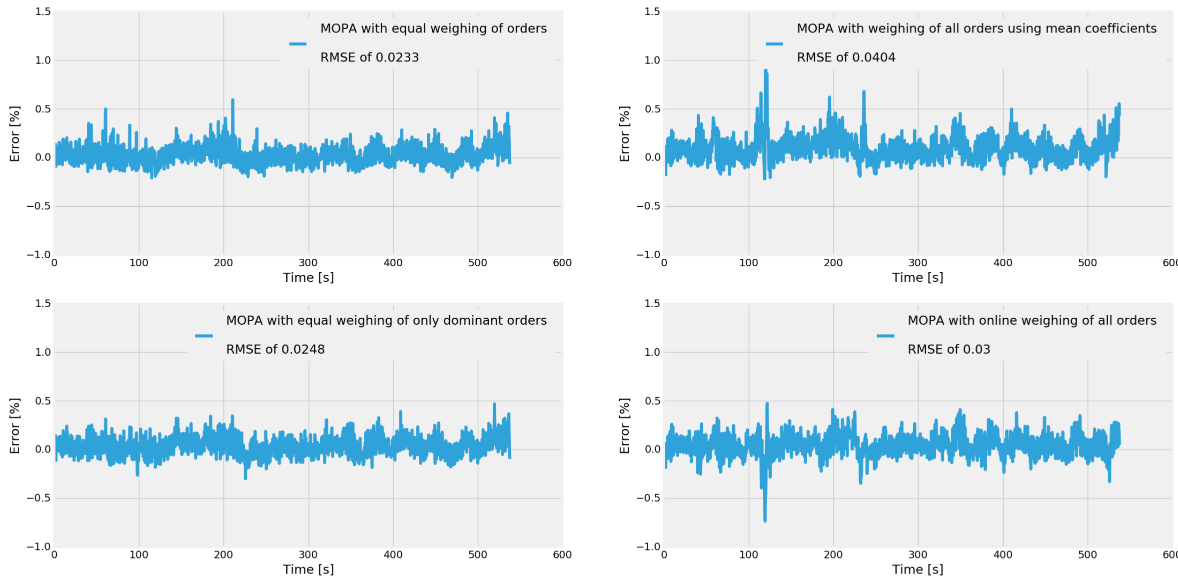


Figure 6: Relative errors between the reference shaft speed and the different approaches for the MOPA method.

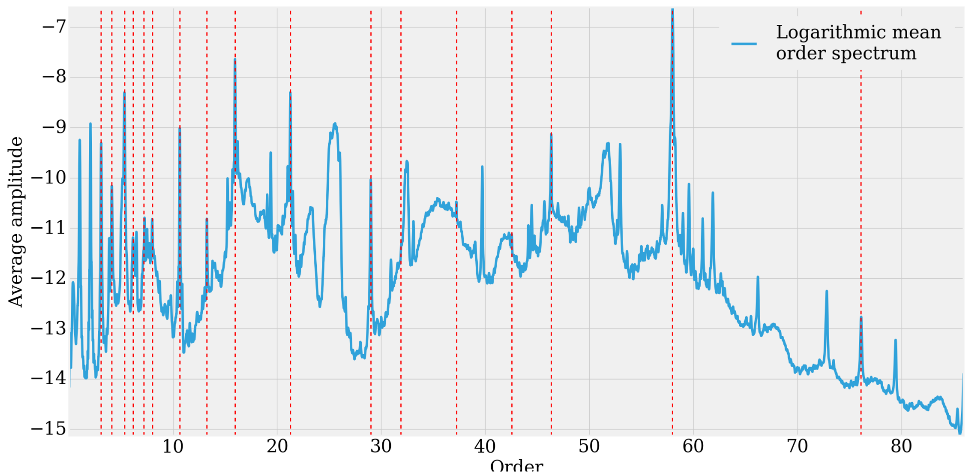


Figure 7: Mean order spectrum used to increase certainty about dominant harmonic orders .

# 4 Discussion & Conclusions

This paper investigates the possibility to optimize the use of the fundamental order selection used as input for the multi-order probabilistic approach. More specifically, it looks into combining the different pdfs, each corresponding to one harmonic order, in an optimized manner before the continuity step to increase the accuracy and decrease the computational efforts. A straightforward least squares approach is used to fit the coefficients for every order in every time step. This leads to a set of time-dependent weights that can be averaged over time to obtain insight in the dominant orders that lie closest to the ideal situation of a pdf containing only one peak related to the shaft speed. Four approaches are examined that make use of these coefficients in a different way. Three of the approaches make use of all harmonic orders, but differ on the weighting of them. The standard MOPA method uses equal weighting of all harmonic orders, the second approach uses the time averaged coefficients as weights for the orders in every time step, and the third uses time dependent weights. The fourth approach uses only a reduced set of dominant orders with equal weighting. It is found that the standard approach and the dominant orders approach perform best in accuracy of tracking the IAS. While all four approaches have acceptable results, it is somewhat surprising that the coefficient weighted approaches perform worse for some time instances, since they utilize the optimization results. Most likely this is due to the least-squares results being skewed for particular time frames due to poor excitation of the shaft speed related harmonics in pdfs with high weight values. From a computational point of view, it is desirable though that the reduced set of orders can produce results of equal quality to the standard MOPA since it can significantly reduces computation time and required memory. Future research will test whether this approach performs similarly well on other data sets. Positive results would for example suggest the use of an initial measurement with a tachometer on a single wind turbine gearbox. This measurement can then be used to determine the optimal set of harmonic orders. Other wind turbines in the farm with identical gearboxes can then use the same order set, multiplying the reduction in computational cost depending on the amount of turbines present in the farm.

## 5 Acknowledgment

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