Bayesian approach in the predictive maintenance policy

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Abstract

In Industry, the maintenance policy is devoted to avoid sudden failures that can cause the stop of the system with a consequent loss of production, or - at least - to the minimization of the failure probability and/or the preservation of this probability under a fixed value. In such systems, the use of sensors for the monitoring of their degradation level is very useful. This gives the possibility to follow the time history of the component and to identify the most appropriate time for the maintenance activities, making possible the exploitation of the component for almost its whole useful life. The traditional preventive maintenance policy makes use of the a priori information on the population by assuming a probability distribution function and by estimating the involved statistical parameters [1]. By a monitoring system further information on the stochastic degradation process of the particular component belonging to the population can be available. Nevertheless, such sensors add new costs and exhibit inaccuracy in tracking the stochastic process. This inaccuracy implies an uncertainty in the supplied information. This occurs whether the degradation is defined as a geometric characteristic of the component or as the exhibition of a particular effect. For example, in a cutting tool, wear changes the geometrical characteristics causing an increase of superficial roughness on the machined parts. If a maximum value of roughness is accepted, the condition of failed cutting tool corresponds to the reaching of such value. In this case, the vibration signal is not a correct fault indicator because it is not suitable for tracking the degradation process. For these reasons, a predictive maintenance policy presupposes the identification of a signal well correlated to the degradation process and a high precision monitoring system. Components whose sudden failure can produce dramatic consequences on the system availability are considered. They must operate with a high required degree of reliability and the maintenance policy must assure a reliability level not lower than a pre-defined value. This paper is the second part of two [2], presenting an algorithm for the implementation of a sensor-driven predictive policy based on a Bayesian approach. Simulation results are supplied.

1 Introduction

For some industrial systems, the main objective of the maintenance activity is the minimization of the failure probability or, at least, the preservation of this probability under a fixed value. In such systems sensors are employed to monitor components degradation process over time [3], [4] with the objective to identify the most appropriate time for the starting of the maintenance actions. This results in a better exploitation of the monitored component during its useful life. With the development of electronic monitoring systems and of modern diagnostic tools, a significant interest for the predictive maintenance [5, 6, 7, 8] policy has increased with respect to the traditional preventive one. Actually, the preventive maintenance policy is exclusively based on the a-priori information of the population without any consideration of the specific behavior of each component. Through a monitoring system further information on the stochastic degradation process of each item can be available both for the prediction of the remaining time to failure [9] and for the scheduling of the optimal maintenance time. Such approach is particularly useful for the technical staff managing the maintenance actions. Actually, it gives the possibility to plan the maintenance resources, to be employed for the plant maintenance activities [10], with an adequate advance. Nevertheless, sensors use add new costs and new uncertainties in the supplied information. These uncertainties occur whether the degradation is defined as a geometric characteristic of the component or as the exhibition of a particular effect. For instance, in a cutting tool, wear changes the geometrical characteristics causing an increase of superficial roughness on the machined parts [11].

If a maximum value of roughness is accepted, the condition of failed cutting tool corresponds to the reaching of such value. In this case, the vibration signal is not the best fault indicator because it can not follow the degradation process accordingly. A second kind of uncertainty depends by the monitoring system itself. A monitoring system is in fact constituted by different parts: the connection between the component and the sensor, the sensor itself, the transmission system, the signal amplifier, the acquisition system.

Therefore, a predictive maintenance policy presupposes the identification of a physical signal well correlated to the degradation process and of a high precision monitoring system. In this paper, components whose sudden failure can produce dramatic consequences on the system availability are considered. Therefore, it is assumed that they must operate with a high required degree of reliability [12] and that the maintenance policy must assure a reliability level not lower than a fixed value. In order to choose between the implementation of the traditional preventive maintenance policy and the predictive one, a comparison is proposed by simulation. Actually, by hypothesizing a degradation model [3, 9, 13], it is shown how the convenience of the predictive maintenance approach depends both on the parameters characterizing the stochastic degradation process and on the uncertainty of the monitoring system. In this paper, the Bayesian approach [14] is proposed to update the a priori information on the population which the components belong to, as an effective and dynamic tool to identify the different behaviors of each component during its useful life. This approach has been proposed in different context. In [15], authors investigate how the use of a Bayesian updating procedure changes the characteristics of the failure rate associated with the time-to-failure distribution. In [16] it is emphasized how the remaining useful life (RUL) can differ for similar components operating under the same conditions. To detect such differences, Authors propose a Bayesian approach for predicting the RUL of critical components. Finally, in [17], a Bayesian approach is proposed to manage a water distribution network based on the evaluation of the reliability of the network components. The paper is organized as follows: section 2 introduces the degradation model and the process monitoring, section 3 presents the Bayesian approach. In section 4, a comparison between the preventive and the predictive maintenance policy is presented. Section 5 reports the simulation results and, finally, in section 6 conclusions are drawn.

2 Modeling the degradation process and process monitoring

The application of the predictive maintenance policy is preceded by the selection and measurement of one or more variables representative of underlying degradation process. In this study a first autoregressive model with drift (AR(1)) or non stationary random walk model (RWM) (see Appendix A) is assumed to model the degradation process on the physical parameter of interest, say y [11]:

$$y(t+dt) = y(t) + \gamma' dt + \varepsilon(t) \tag{1}$$

In equation 1, γ' is the drift, $\varepsilon(t)$ is a white process normally distributed with zero-mean and variance σ_ε^2. Since the degradation process is observed at regular times Δt (unit of time), model (1) will be discretized. By setting $\gamma = \int \gamma' dt = \gamma' \Delta t$, equation 1 becomes:

$$y_{i+1} = y_i + \gamma + \varepsilon_{i+1} \tag{2}$$

The degradation path cannot be generally observed directly but through a monitoring system that supplies a variable m correlated to the real degradation path y. If m_i represents the value of such variable at time t_i and the output of the monitoring system is linear, the relation between m_i and y_i can be expressed by:

$$m_i = a + by_i + \delta_i \tag{3}$$

where a e b are the coefficients of the linear transformation and δ_i is the total system error at time i. Let δ be normal distributed with zero-mean and variance δ_γ^2. From equation 3, y_i can be expressed as follows:

$$y_i = \frac{m_i - a - \delta_i}{b} \tag{4}$$

By substituting equation 4 in equation 2, it follows:

$$\frac{m_{i+1} - a - \delta_{i+1}}{b} = \frac{m_i - a + \delta_i}{b} + \gamma + \varepsilon_{i+1} \tag{5}$$

Therefore,

$$m_{i+1} = m_i + b\gamma + \Delta_{i+1} + b\varepsilon_{i+1} \tag{6}$$

Where $\Delta_{i+1} = \delta_{i+1} - \delta_i$. Setting $\lambda_{i+1} = \Delta_{i+1} + b\varepsilon_{i+1}$, equation 6 becomes:

$$m_{i+1} = m_i + b\gamma + \lambda_{i+1} \tag{7}$$

where $\lambda \sim N(0, 2\sigma_\delta^2 + b\sigma_\varepsilon^2)$.

3 Bayesian approach

One of the most promising crossover between statistics and machine diagnostics is given by the application of the Bayesian inference to the condition monitoring [18, 19]. The Bayesian inference is a method of statistical inference based on the Bayes' theorem [14], which is used to update the probability for a hypothesis as more evidence or information becomes available. In equation 7, γ, the mean drift characterizing the degradation pattern of the components, is a known deterministic parameter, while λ represents the stochastic behavior of the degradation process. The drift γ represents a constant physical phenomenon common to all the units belonging to the same population. However, each component belonging to a population can exhibit a different degradation behavior depending both on the stochastic nature of the degradation process and on specific aspects, i.e. geometric or metallurgical characteristics or on different environmental working conditions. For these reasons, parameter γ should be more properly considered as a stochastic variable and its value for a specific component as an outcome. In this paper a normal probability distribution with mean μ_γ and variance σ_γ^2 has been assumed for variable γ.

Therefore, equations introduced in the previous section should be referred to each component by specifying it with an index j. With these assumptions, the distribution of the stochastic variable m_{i+1}, conditioned by the acquisition of m_i, is normal with mean and variance given by the following equations:

$$E[m_{i+1}] = m_i + b\mu_{i,j} \tag{8}$$

$$Var[m_{i+1}] = b^2\sigma_{\gamma,j}^2 + \sigma_\lambda^2 \tag{9}$$

Using data coming from the monitoring system, it is possible to estimate more accurately the degradation parameter γ_j for the j-th component. As underlined before, such drift can be considered as an outcome of the stochastic variable γ. The initial available information on this outcome is the distribution of γ, $\pi(\gamma)$ with mean μ_γ and variance σ_γ^2. By a Bayesian approach [14], the a-priori information $\pi(\gamma_j) \equiv \pi(\gamma)$ will be updated on the basis of the acquired data coming from the monitoring system. Therefore, if m_i represents the last acquisition of the monitoring system, the a-posteriori distribution of γ_j for the component j, $p(\gamma \mid m_i)$, is normal with mean $\mu_{i,j}$ and variance $\sigma_{\gamma,j}^2$. For sake of simplicity, by setting $a = 0$ and $b = 1$ in equation 3, mean and variance of variable γ can be updated through the following equations [13]:

$$\mu_{\gamma,j} = \frac{\sigma_\gamma^2 m_i + \sigma_\lambda^2 \mu_\gamma}{t_i\sigma_\gamma^2 + \sigma_\lambda^2} \tag{10}$$

$$\sigma_{\gamma,j}^2 = \frac{\sigma_\gamma^2 \sigma_\lambda^2}{t_i\sigma_\gamma^2 + \sigma_\lambda^2} \tag{11}$$

The estimate ($\widehat{m}_{i+1,j}$) of m for component j at time $i+1$, conditioned by the last acquisition at time i, can be drawn from equation 7 as follows:

$$\widehat{m}_{i+1,j} = m_{1,j} + b\gamma_j + \lambda_{i+1,j} \tag{12}$$

$$\widehat{m}_{i+2,j} = m_{i+1,j} + b\gamma_j + \lambda_{i+2,j} = m_{i,j} + 2b\gamma_j + \lambda_{i+1,j} + \lambda_{i+2,j} \tag{13}$$

By iterating the procedure at time $i+k$:

$$\widehat{m}_{i+k,j} = m_{i,j} + kb\gamma_j + \lambda_{i+1,j} + \lambda_{i+2,j} + \cdots + \lambda_{i+k,j} \tag{14}$$

The estimate of mean and variance of $\widehat{m}_{i+k,j}$, are:

$$\widehat{\mu}_{i+k,j} \mid_{m_{i,j}} = m_{i,j} + bk\mu_{\gamma,j} \tag{15}$$

$$\widehat{\sigma}^2_{i+k,j} \mid_{m_{i,j}} = b^2k^2\sigma^2_{\gamma,j} + k\sigma^2_{\lambda} \tag{16}$$

4 Comparison between preventive and predictive maintenance

The described monitoring system can be used for the implementation of a predictive maintenance policy. The maintenance policy must assure a reliability for the monitored component not lower than a fixed value. Let $f(\tau)$ be the failure time distribution of the population which the component belongs to and $F_\tau(t*)$ the unreliability of such component at time $t*$. It follows $F_\tau(t*) = P\{\tau \le t*\}$, i.e. the probability that the component failure takes place before $t*$.The monitored component will be considered failed when the degradation level will reach a limit value $y*$ that corresponds to a threshold value $m*$ for the monitored parameter. Thus, the probability that the component fails before $t*$ is the probability that the degradation signal $m(t*)$ at time $t*$ is equal or greater than the threshold $m*$. Consequently, $F_\tau(t*) = P\{\tau \le t*\} = P\{m(t*) \ge m*\}$. Reliability $R(t*)$ is obviously $1 - F_\tau(t*)$ [12]. If T indicates the necessary time to plan the maintenance activities, it is possible to estimate, at the acquisition time t, the degradation level of the monitored component at the future time $t+T$. Assuming T as an integer multiple of the acquisition interval t, the reliability of the generic component j at time $t+T$, can be computed as follows:

$$R_j(t+T) = \int_{-\infty}^{m*} f(\widehat{m}_{t+T})d\widehat{m} \tag{17}$$

$$= \int_{-\infty}^{m*} \frac{1}{\sqrt{2\pi}\sqrt{b^2T^2\widehat{\sigma}^2_{\gamma,j} + T\sigma^2_\lambda}} \{-\frac{1}{2(b^2T^2\widehat{\sigma}^2_{\gamma,j} + T\sigma^2_\lambda)}[\widehat{m}_{t+T} - (m_t + bT\widehat{\mu}_{\gamma,j})]^2\}d\widehat{m}_{t+T} \tag{18}$$

To show how the variables characterizing the degradation process and the monitoring system can influence the advantages achievable with the implementation of the predictive maintenance policy, the degradation process of 3.000 components, following the stochastic process described in the previous section, was simulated with Matlab. In order to estimate the a-priori information on the mean and variance (μ_γ and σ^2_γ) of the population, the degradation behavior of 25 components was observed by simulation.

These a-priori information was used for the determination of the failure time probability distribution parameters and for the evaluation of the maintenance time interval (t_{prev}), when a preventive maintenance policy is employed. The same a-priori information is updated according to equation 10 and 11 by Bayesian approach when a predictive maintenance policy is adopted (t_{pred}).

Considering the assumptions on the stochastic process, the distribution function of the failure time, under the condition that the threshold $m*$ has been reached, can be obtained by the Bayes formula as an a-posteriori probability. This distribution is known as inverse Gaussian [11]. From this distribution, it is possible to calculate the replacement interval respecting the constraint on reliability or equivalently on the accepted risk. The same constraint on reliability is considered for the calculation of the replacement instants, when the predictive maintenance policy is adopted. In both cases failure risk of 0.01 (1%) is considered. In the case of predictive policy, T is fixed to 10, i.e. it is assumed that 10 time unities are sufficient to plan the maintenance activities.

It is also assumed that the scheduled time for the starting of the maintenance activities can be changed only if a failure occurs in the time interval $[t, t+T]$.

For each simulated component, the calculated times t_{pred} and t_{prev} are compared with the actual time (t_{real}) corresponding to the reaching of the fixed threshold $m*$.

In particular, the effectiveness of the two maintenance policies is here evaluated by calculating the distance between the couples $t_{pred} - t_{real}$ and $t_{prev} - t_{real}$. The smaller the relative mean distance, the better the maintenance policy is. Actually a smaller time distance corresponds to a better exploitation of the component. To these purposes, the following equations are employed:

$$d_{pred} = \frac{1}{N} \sum_{j=1}^{N} \frac{|t_{pred,j} - t_{real,j}|}{t_{real,j}} \tag{19}$$

$$d_{prev} = \frac{1}{N} \sum_{j=1}^{N} \frac{|t_{prev,j} - t_{real,j}|}{t_{real,j}} \tag{20}$$

5 Simulation results

By choosing different combinations of the variances σ_γ^2, σ_ε^2, σ_δ^2, the relative mean distances were computed according to equations 19 and 20. Results are shown in Table 1.

Simulation	σ_γ^2	σ_ε^2	σ_δ^2	d_{pred}	d_{prev}
1	0.4	0.4	0.2	0.1835	0.2787
2	0.4	0.2	0.2	0.1804	0.2749
3	0.4	0.4	0.4	0.1896	0.2787
4	0.2	0.2	0.2	0.1813	0.2272
5	0.2	0.2	0.6	0.1945	0.1925
6	0.2	0.4	0.4	0.1891	0.1925
7	0.2	0.4	0.8	0.1995	0.1925
8	0.2	0.4	0.2	0.1834	0.1925

Table 1: Mean relative distances between times obtained by simulation

Results reported in Table 1 show the influence of different sources of uncertainty on the adopted maintenance policy. Although no experimental plan was employed, it is possible to draw some interesting considerations. The predictive maintenance policy appears little influenced by the variability of the parameter γ, while it is more influenced by the variability of the degradation model σ_ε^2 and by the uncertainty of the monitoring system σ_δ^2 . Different considerations can be done for the preventive maintenance policy. Actually, it appears more influenced by the variability of the parameter γ and little by σ_ε^2. Obviously it is not influenced by σ_δ^2 because no monitoring system is employed. In conclusion, the predictive maintenance policy is always more effective than the preventive one, excluding those cases with the highest value of σ_δ^2. The advantages of the predictive policy increases significantly with the increasing of the parameter σ_γ^2, is less influenced by the variability of the degradation process and decreases with the increasing of the uncertainty of the monitoring system, until this policy can become not convenient.

6 Conclusions

In this paper, a comparison between the traditional preventive maintenance policy and the predictive one is presented by simulation. A Bayesian approach is proposed for the integration between information coming from a monitoring system and the a-priori knowledge of the degradation process when a predictive maintenance policy is adopted. The proposed degradation model takes into account different sources of variability and their impact on the adopted maintenance policy (preventive vs predictive) is presented. In particular, it is shown how the predictive maintenance policy makes possible a better exploitation of a component and how this advantage can gradually be lost with the increasing of the uncertainty of the monitoring system. The proposed procedure can be used for an early estimation about the convenience in the implementation of a predictive maintenance policy, especially in those real situations where system reliability is the most critical parameter.

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A Appendix

A Random Walk Model (RWM) is a non stationary model. In a model with drift, y_{i+1} is linked to y_i by the following equation: $y_{i+1} = y_i + \gamma + \varepsilon_{i+1}$ where γ is the drift and ε_{i+1} a white noise process, $\varepsilon_{i+1} \sim N(0, \sigma^2)$.

It is possible to write $y_1 = y_0 + \gamma + \varepsilon_1$, $y_2 = y_1 + \gamma + \varepsilon_2$, $y_3 = y_2 + \gamma + \varepsilon_3$, etc...

Therefore, $y_3 = y_0 + 3\gamma + \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ and generally $y_k = y_0 + k\gamma + \sum \varepsilon_k$.

It results: $E(y_k) = y_0 + k\gamma$ and $Var(y_k) = k\sigma^2$. Hence, in a RWM with drift, both its mean and variance increase over time such that it is a non stationary process.